

Part 1 Business Analysis

Section D. Quantitative Methods (Levels A and B)
15% of Part 1.

PREPARED BY

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Decision-Making and Quantitative Methods

- **Forecasting analysis**
 - ✓ Regression analysis
 - ✓ Learning curve analysis
 - ✓ Exponential smoothing
 - ✓ Time series analysis
- **Linear programming**
 - ✓ Scarce resource considerations
 - ✓ Capacity constraints
- **Network analysis**
 - ✓ Critical Path Method (CPM)
 - ✓ Program Evaluation Review Technique (PERT)

Decision-Making and Quantitative Methods

- **Probability concepts**
 - ✓ Probability & Expected value
- **Decision tree analysis**
 - ✓ Assumptions of decision tree analysis
 - ✓ Estimating cash flow and probability values
- **Other quantitative techniques**
 - ✓ Sensitivity analysis
 - ✓ Simulation
 - ✓ Queueing theory
 - ✓ Markov process

Quantitative Methods in the CMA exam

- This is one of the smaller sections and one that you do not want to spend a significant amount of time preparing for.
- Candidates should expect about 17 questions on quantitative methods in Part 1 of the exam.
- Experience has shown that some of the questions are very basic and could be answered by many people without any preparation.
- At the other end of the spectrum, there are a few questions that will be very involved and, without a thorough preparation of this topic, you will not be able to answer them.
- It is mostly for the middle group of questions that you need to prepare.
- Your preparations should not be focused on detailed numerical calculations and applications of the methods discussed. Rather, you need to be certain that you understand what the different tools and methods are (for example, linear regression or PERT), what they are used for and the basics of how they work.

Forecasting

Causal forecasting methods (Regression Analysis)
Time Series methods

Forecasting

- ➔ Forecasting is a critical part of any business, and it involves looking into the future and attempting to determine what future conditions and/or results will be. A budget is a form of forecasting.
- ➔ Forecasting Methods
 - ✓ Quantitative Methods.
 - Causal forecasting methods (Regression Analysis)
 - Time Series methods
 - Smoothing (Moving Average, Weighted Moving Average, Exponential smoothing)
 - Trend Projection
 - Trend projection adjusted for seasonal influence
 - ✓ Qualitative Methods
 - Delphi Method
 - Expert judgment
 - Scenario Writing

Time Series Methods

Smoothing

(Moving Average, Weighted Moving Average, Exponential Smoothing)

Time Series Methods

- **Time series data** reflects activity for one variable – an organization, plant, activity, or one expense classification – over a sequence of past time periods.
- **A time series method of forecasting** uses only these historical values in an attempt to find (discover) a pattern in them that can be used in forecasting the future. Only that one set of historical time series data is used in time series analysis and that historical data is not compared to any other set of data.

Components of the Time Series

- A time series may have one or more of four **patterns** (also called components) that influence its behavior over time:
- ✓ **Trend or secular trend** (Resulting from long term- multiyear -factors)
 - ✓ **Cyclical** (Resulting from long term- multiyear- cyclical movements in the economy)
 - ✓ **Seasonal** (Resulting from factors within one year or one day)
 - ✓ **Irregular or Random Variation** (Resulting from short term, unanticipated ,and nonrecurring factors)

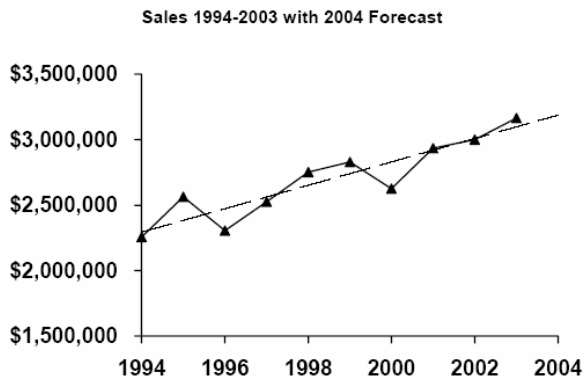
Trend Pattern

- Over a long period of time, the historical data may exhibit a **trend**, which is a gradual shifting to a higher or lower level.
- If a long-term trend exists, there will probably still be short-term fluctuations within that trend; however, the long-term trend will be apparent and the later usually the result of long term factors such as changes in the population , demographic characteristics of the population , technology , and consumer preferences.

Trend Pattern

- For example, sales from year to year may fluctuate but overall, they may be going up, as is the case in the graph below.

➤ The long-term sales trend has been upward from 1994 to 2003, despite the dips in 1996 and 2000. According to this trend, a reasonable sales forecast for 2004 would be \$3,250,000.



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Cyclical Pattern

- A long-term trend line can still be established even if the sequential data fluctuates greatly from year to year due to cyclical factors. Any recurring fluctuation that lasts longer than one year is attributable to the **cyclical component** of the time series.
- The cyclical component is usually due to the cyclical nature of the economy.
 - ✓ For example: periods of modest inflation followed by periods of rapid inflation can lead to many time series that alternate below and above a generally increasing trend line.

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Seasonal Pattern

- In order to identify trend and cyclical components of a time series, we track the annual historical movements of the data over several years. That is, we look only at results for full years, such as total sales for the years 1994 through 2003.
- However, a time series can fluctuate within the year due to seasonality in the business.
 - ✓ For example, a manufacturer of swimming pools expects low sales activity in winter months, with peak sales in summer.
- The Seasonal component also may be used to represent any regularly repeating pattern that is less than one year.
 - ✓ For example, daily traffic volume data show within the day "seasonal" behavior, with peak levels during rush hours, moderate flow during the rest of the day, and light flow from midnight to early morning.

Seasonal Pattern (cont'd)

- Seasonal variations are common in many businesses. **In practice**, a variety of methods exist for including seasonal variations in a forecasting model, but most methods use a seasonal index.
- Alternatively, seasonal variations can be removed from data by using a **weighted average of several time periods** instead of data from individual periods.

Irregular Pattern

- A time series can also vary in a random pattern, not repeating itself in any regular pattern.
- This is called the **irregular pattern or random variables**. It is caused by short-term, nonrecurring factors, and its impact on the time series cannot be predicted.
 - ✓ For example , business can be affected by random happenings (e.g., weather, strikes, fires, etc.).

Using Time Series methods

- The objective of time series analysis is to develop a forecast for future results.
- Time series methods are used in forecasting in three ways:
 - ✓ **Smoothing** (moving averages, weighted moving averages and exponential smoothing),
 - ✓ **Trend projection**, and
 - ✓ **Trend projection adjusted for seasonal influence**.

SMOOTHING

- Just as the name implies, smoothing methods attempt to “smooth out” random fluctuations caused by the **irregular component** of a time series.
- Smoothing methods work with a time series that has **no significant trend, cyclical or seasonal effects**. They do not work well when there is a long-term upward or downward trend or when there is cyclical variation or seasonal variation.
- However, when nothing affects the values except random variations, smoothing methods can provide highly accurate, short-range forecasts such as a forecast for the next time period .

Moving Averages

- Use the average of the most recent data in the time series. Whenever a new value becomes available for the time series, it replaces the oldest value.
 - ✓ For example, when using a four-week moving average to forecast sales, to forecast sales for week five, we would average the sales for weeks one through four. The forecast of sales for week ten would use the average sales for weeks six through nine.

Weighted Moving Average

➤ Is a variation of the moving average method. When utilizing this method, we use different **weights** for each value and compute a weighted moving average, using the most recent data in the time series.

- ✓ For example, we might give more recent historical values weights that are greater than those given to the older values. If there is four months of data, to forecast the fifth month's value using a weighted moving average, we would approach it in the manner outlined in the following example.

Example

➤ ABC Corporation wants to use a four-month weighted moving average method to forecast sales for the month of May. Actual sales for ABC for the months of January, February, March and April are as follows:

January	\$21,000,000
February	23,000,000
March	25,000,000
April	20,000,000

Example (cont'd)

- ABC has assigned a descending weight to each month's values, starting with the most recent month. Each of the month's results is multiplied by the weight, and then these individual monthly values are added together to determine the May forecast. The weights ABC has assigned to the four previous months are 40%, 30%, 20% and 10%. This means that the results in the most recent month (April) will have four times the impact on the May forecast as the oldest month (January).
- Note that the total of all the weights equals 10/10, or 1.
- The weighted moving average is the total, \$22,200,000, and this is the expected result for the month of May.

			<u>Weight</u>			
April	\$20,000,000	*	4/10	=		\$ 8,000,000
March	25,000,000	*	3/10	=		7,500,000
February	23,000,000	*	2/10	=		4,600,000
January	21,000,000	*	1/10	=		<u>2,100,000</u>
						<u>\$22,200,000</u>

Exponential Smoothing

- Is a special type of weighted moving average. With exponential smoothing, we forecast a value for the next period by calculating a weighted average of two numbers only:
 - ✓ The most recent period's **actual** value, and
 - ✓ The most recent period's **forecasted** value, using exponential smoothing.

Exponential Smoothing (cont'd)

➔ The calculation is expressed algebraically as follows:

$$F_{t+1} = \alpha Y_t + (1 - \alpha) F_t$$

Where:

- F_{t+1} = forecast for the next period
- Y_t = actual value for period t
- F_t = forecasted value for period t
- α = smoothing constant (between 0 and 1)

Example

➔ Example: In January, ABC Corporation began using exponential smoothing to forecast sales for each month. Actual and forecasted sales, in millions, for ABC for the months of January, February, March and April are as follows. Forecasted sales for January through April have been calculated using exponential smoothing and an *alpha* of .1.

	Actual (Y)	Forecasted* (F)
January	\$21.0	N/A
February	23.0	\$21.0
March	25.0	21.2
April	20.0	21.6

*Forecasted by means of exponential smoothing.

Example (cont'd)

- To calculate a sales forecast for the month of May using exponential smoothing, use only the **actual sales for the month of April** and the **forecasted sales (forecasted using exponential smoothing) for the month of April**.
- As you may have noticed, not just any forecast can be used in this calculation. For exponential smoothing to work, the forecasted value used can only be one that was calculated using exponential smoothing.

Example (cont'd)

- The forecasted sales figure for the month of May will be:

$$F_{t+1} = (.1 * 20) + (.9 * 21.6)$$

$$F_{t+1} = 21.4$$

Causal Forecasting

**Regression Analysis
High & Low Method**

Causal Forecasting

- Causal forecasting methods are used when the value that we are forecasting can be determined to be affected by some other value.
- If we can identify a **cause and effect relationship** between what we are forecasting and the other value, and if that relationship is a linear one, we can use a projection of the other value to forecast the value we are interested in.

Regression Analysis

Regression analysis is a statistical method for obtaining the cost estimation unique equation that best fits a set of data points.



Least squares regression is widely viewed as one of the most effective methods for estimating costs.

Regression Analysis

Statistics courses and computer courses deal with detailed regression computations using computer spreadsheet software.



Accountants and managers must be able to interpret and use regression estimates.

Two basic assumptions of simple Regression Analysis

- 1) Changes in the value of the dependent variable can be explained by changes in the level of the independent variable; and
- 2) The relationship between the dependent variable and the independent variable is linear. That is, a graph of the two variables, with the independent variable on the x-axis and the dependent variable on the y-axis, will result in a straight line within the relevant range.

Example

- For example, if we know that the level of sales is directly related to the level of advertising, we can forecast sales once we know the future level of advertising. Therefore, once the advertising budget has been set, we can use planned advertising expenditures to forecast sales.
- How do we know whether or not there is a relationship between advertising expenditures and sales? We use regression analysis, as described in the preceding section. The only difference is that instead of using years or quarters as the independent variable, we use advertising expenditures during each year or quarter as the independent variable .

Example (cont'd)

- We use historical data on advertising expenditures (on the x axis, the independent or predictor variable) and sales (on the y axis, the dependent or response variable), graph it and do correlation analysis to determine whether there is a linear relationship between the two variables.
- Note: In order to use regression analysis, there must be a reasonable basis to expect the dependent variable to be caused by the independent variable. If there is no reason for a connection, any connection found through the use of regression analysis is accidental. **So we must be careful not to assume that a linear relationship means there is a cause and effect relationship.**

Simple Linear Regression vs. Multiple Regression Analysis

- **Simple Linear Regression**
 - ✓ If there is only one independent variable and one dependent variable and the relationship between them is linear, regression analysis is called **simple linear regression**, just as it was called in the previous example.
- **Multiple Regression Analysis**
 - ✓ However, it is also possible for one dependent variable (for example, sales) to be affected by more than one independent variable (for example, advertising expenditures, size of the sales staff, competition, the economy and any number of other possible causes). When there is more than one independent variable, the regression analysis is called **multiple regression analysis**.

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Regression Analysis

The objective of the regression method is still a linear equation to estimate costs $Y = a + bX + e$

Y = value of the dependent variable, estimated cost

a = a fixed quantity, the intercept, that represents the value of Y when X = 0

b = the unit variable cost, the coefficient of the independent variable measuring the increase in Y for each unit increase in X

X = value of the independent variable, the cost driver

e = the regression error, the distance from the regression line to the data point (actual)

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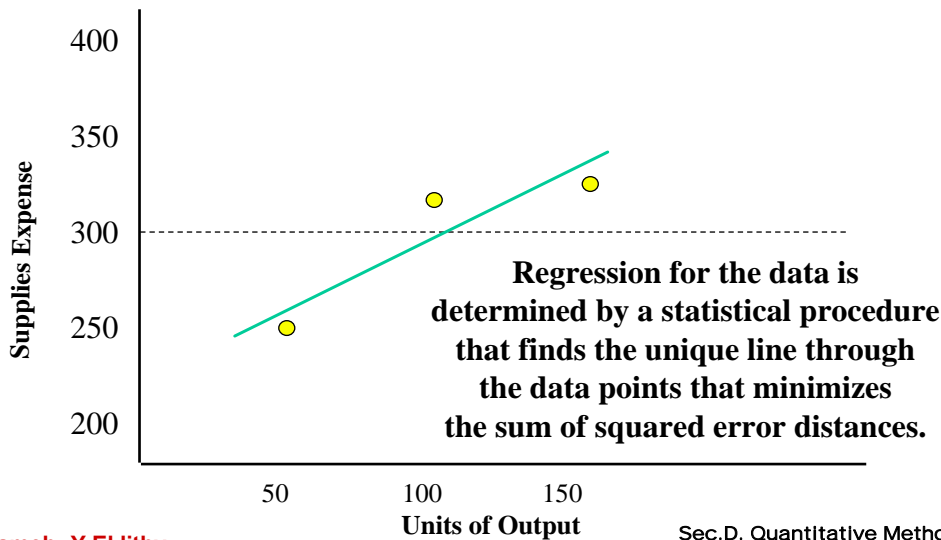
Regression Analysis

<u>Month</u>	<u>Supplies Expense (Y)</u>	<u>Production Level (X)</u>
1	\$250	50 units
2	310	100 units
3	325	150 units
4	?	125 units

Regression analysis will enable us to predict the amount of supplies expense for month four.

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Regression Analysis



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Regression Analysis

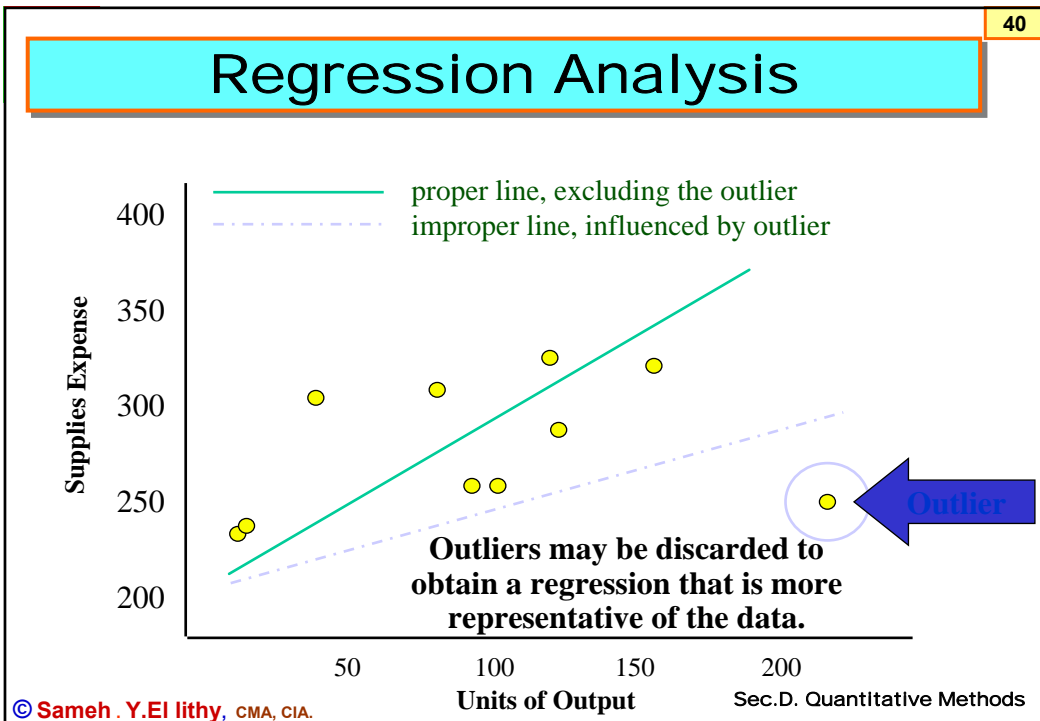
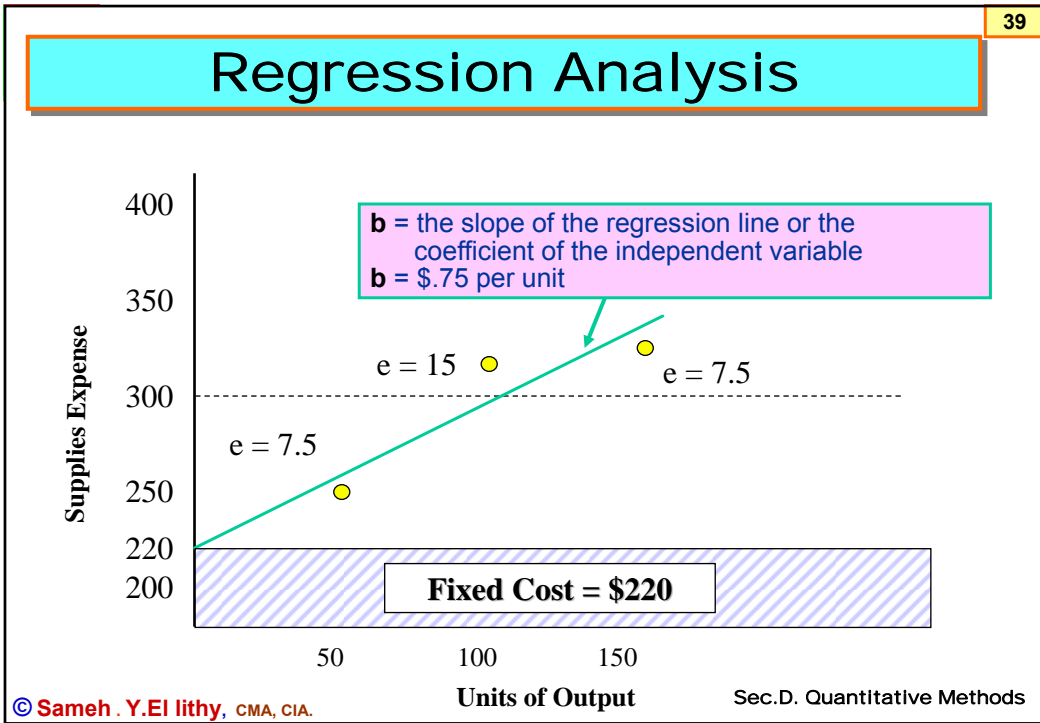
<u>Month</u>	<u>Supplies Expense (Y)</u>	<u>Production Level (X)</u>
1	\$250	50 units
2	310	100 units
3	325	150 units
4	?	125 units

$Y = a + bX + e$
 $Y = \$220 + \$0.75 \text{ per unit} \times 125 \text{ units}$
 $Y = \$313.75 \leftarrow \text{Expense estimate for month 4}$

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Regression Analysis: Measuring Precision and Reliability

➔ To determine whether there is a linear relationship between the independent variable (s) under consideration, we calculate the **coefficient of correlation r** and the **coefficient of determination r^2** as we did when working with one variable (sales) over time.

➔ R

- ✓ The **coefficient of correlation** measures the relative strength of the linear relationship.
- ✓ Takes value $\{-1.0 < r < 1.0\}$.
 - ➔ A value of -1.0 indicates a perfectly inverse linear relationship between x and y .
 - ➔ A value of zero indicates no linear relationship between x and y .
 - ➔ A value of $+1.0$ indicates a direct relationship between x and y .

Regression Analysis: Measuring Precision and Reliability

➔ R-squared

- ✓ A number between zero and one that describes the explanatory power of the regression (the degree to which the change in Y can be explained by changes in X)
- ✓ A relative measure of “goodness-of-fit” (i.e., the percentage change in Y that can be explained by changes in X)
- ✓ The maximum value for R^2 is 1.00 (i.e., 100%)
- ✓ If the coefficient of determination, or r^2 , is low, it may mean that we are using the wrong independent variable in our analysis. The coefficient of determination (r^2) measures the percentage of the **total variance** in cost that can be explained by the regression equation.

Regression Analysis: Measuring Precision and Reliability (continued)

➔ T-value

- ✓ A measure of the statistical reliability of each independent variable in the cost function: does the independent variable have a valid, stable, relationship with dependent variable?
- ✓ Variables with a low *t*-value should be evaluated and possibly removed to improve cost estimation
- ✓ The **t-value** measures the **reliability** of each independent variable, which is **the degree to which the independent variable has a valid, long-term relationship with the dependent variable**. The *t*-value should generally be greater than 2. A value below 2 indicates little or no relationship between the two variables and thus, that independent variable should not be used for causal forecasting.

Regression Analysis: Measuring Precision and Reliability (continued)

➔ Standard error of the estimate (SE)

- ✓ A measure of the accuracy of the regression's estimate
- ✓ An *absolute* measure of "goodness-of-fit" for the regression equation (i.e., SE measures the average variability of the data points around the regression line; an SE of zero means that all of the data points are on the regression line)
- ✓ Related computationally to R^2 (an SE of 0 implies an R^2 of 100%)
- ✓ SE can be compared to the average size of the dependent variable
 - ➔ If the SE value is relatively small compared to the value of the dependent variable, the regression model can be viewed as relatively "good"

Regression Analysis

Evaluating a Regression Analysis

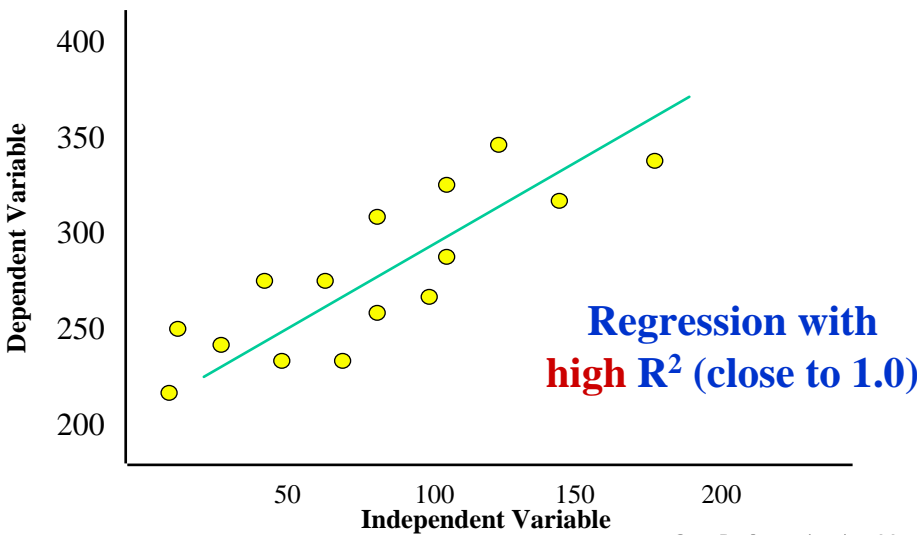
R^2 , the **coefficient of determination**, is a measure of the explanatory power of the regression, the degree that changes in the dependent variable can be predicted by changes in the independent variable.

The **t-value** is a measure of the reliability of each of the independent variables.

The **standard error of the estimate (SE)** is a measure of the accuracy of the regression's estimates.

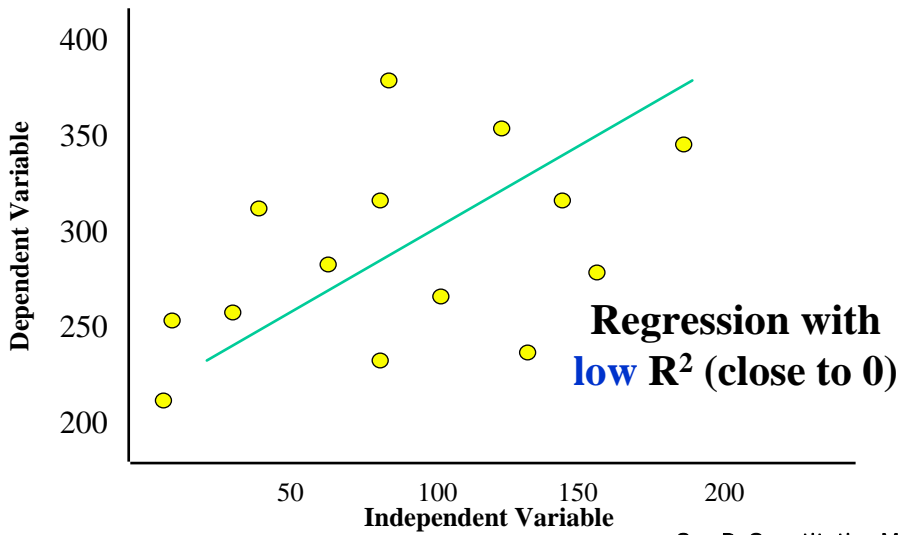
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Regression Analysis



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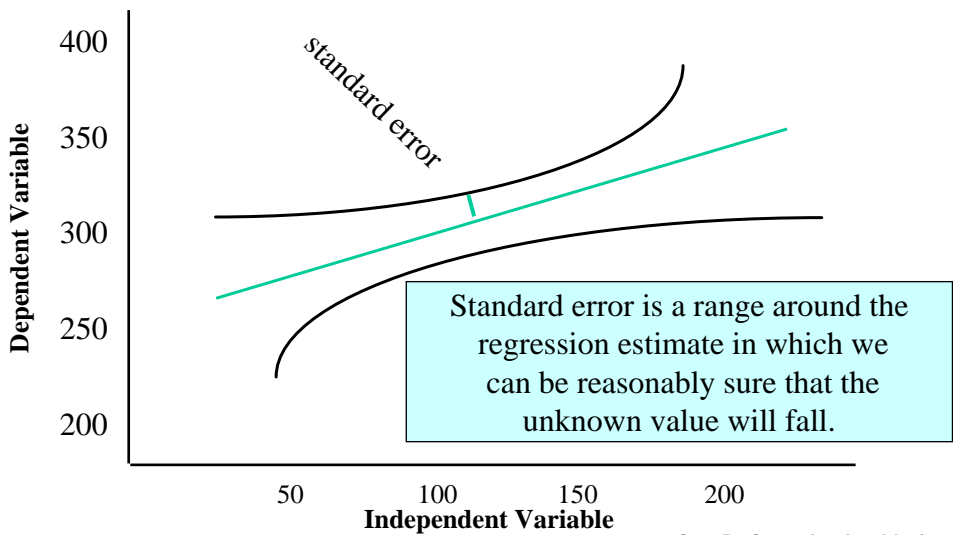
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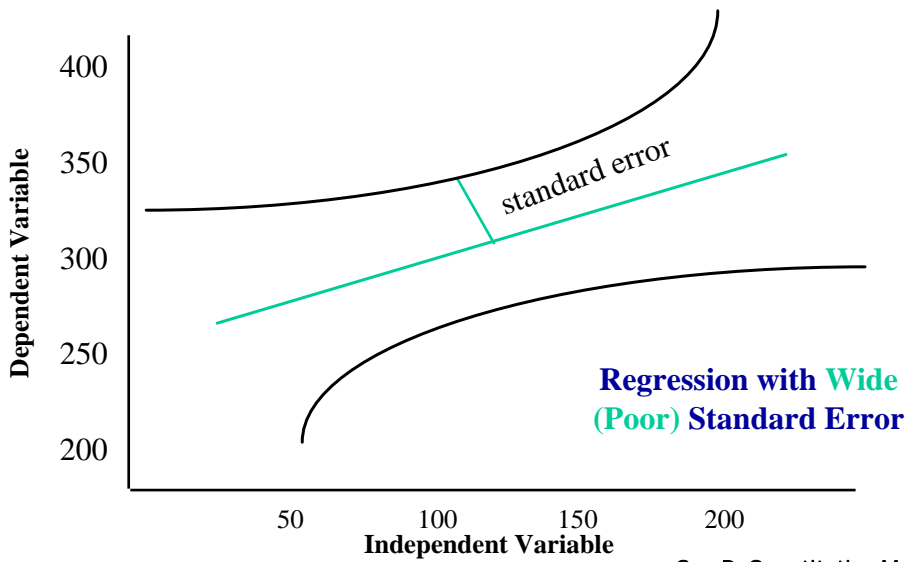


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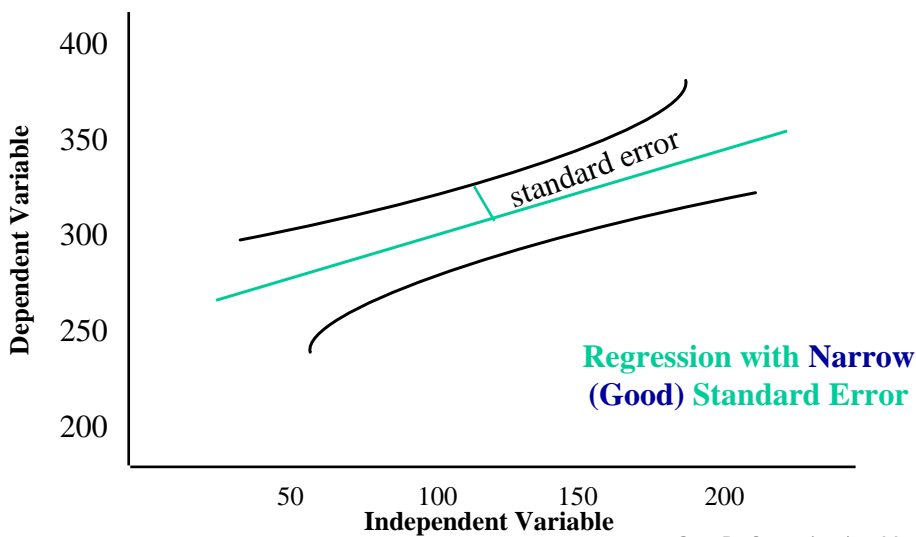
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Regression Analysis



Regression Analysis



Benefits and Limitations of Regression Analysis

- The benefits or advantages of regression analysis are:
 - ✓ Regression analysis is a quantitative method and as such, it is objective. A given data set generates a specific result. That result can be used to draw conclusions and make forecasts.
 - ✓ Thus, regression analysis is an important tool for use in budgeting and cost accounting. In budgeting, it is virtually the only way to compute fixed and variable portions of costs that contain both fixed and variable components (**mixed costs**).

Benefits and Limitations of Regression Analysis (cont'd)

- The shortcomings or limitations of regression analysis are:
 - ✓ To use regression analysis, historical data is required for the variable that we are forecasting or for the variables that are causal to this variable. If historical data is not available, regression analysis cannot be used.
 - ✓ Even when historical data is available, if there has been a significant change in the conditions surrounding that data, its use is questionable for predicting the future.
 - ✓ analysis are valid only for the range of data in the sample

Benefits and Limitations of Regression Analysis (cont'd)

- ✓ In causal forecasting, the usefulness of the data generated by regression analysis depends upon the choice of independent variable (s). If the choice of independent variable (s) is inappropriate, the results can be misleading.
- ✓ The statistical relationships that can be developed using regression

The high-low method

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The high-low method

- The **high-low method** is used to generate a regression line by basing the equation on only the highest and lowest of a series of observations.
- **EXAMPLE:** A regression equation covering electricity costs could be developed by using only the high-cost month and the low-cost month. If costs were \$400 in April when production was 800 machine hours and \$600 in September when production was 1,300 hours, the equation would be determined as follows:

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The high-low method (cont'd)

- | | | |
|--------------|-----------|-------------|
| ➤ High month | \$600 for | 1,300 hours |
| ➤ Low month | 400 for | 800 hours |
| ➤ Increase | \$200 | 500 hours |
- ✓ Because costs increased \$200 for 500 additional hours, the variable cost is \$.40 per machine hour. For the low month, the total variable portion of that monthly cost is \$320 (\$.40 × 800 hours). Given that the total cost is \$400 and \$320 is variable, the remaining \$80 must be a fixed cost. The regression equation is $y = 80 + .4x$.
 - The major criticism of the high-low method is that the high and low points may be abnormalities not representative of normal events.

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Quick Check ✓

- The correlation coefficient that indicates the weakest linear association between two variables is
- A. -0.73
 - B. -0.11
 - C. 0.12
 - D. 0.35

Quick Check ✓

- Answer (B) is correct. The correlation coefficient can vary from -1 to +1. A -1 relationship indicates a perfect negative correlation, and a +1 relationship indicates a perfect positive correlation. A zero correlation coefficient would indicate no association between the variables. Therefore, the correlation coefficient that is nearest to zero would indicate the weakest linear association. Of the options given in the question, the correlation coefficient that is nearest to zero is -0.11.

Quick Check ✓

- ➔ Correlation is a term frequently used in conjunction with regression analysis, and is measured by the value of the coefficient of correlation, r . The best explanation of the value r is that it
- A.** Is always positive.
 - B.** Interprets variances in terms of the independent variable.
 - C.** Ranges in size from negative infinity to positive infinity.
 - D.** Is a measure of the relative relationship between two variables.

Quick Check ✓

- ➔ Answer (D) is correct. The coefficient of correlation (r) measures the strength of the linear relationship between the dependent and independent variables. The magnitude of r is independent of the scales of measurement of x and y . The coefficient lies between -1.0 and $+1.0$. A value of zero indicates no linear relationship between the x and y variables. A value of $+1.0$ indicates a perfectly direct relationship, and a value of -1.0 indicates a perfectly inverse relationship.

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Quick Check ✓

- What coefficient of correlation results from the following data?

X	Y
1	10
2	8
3	6
4	4
5	2

- A. 0
B. -1
C. +1
D. Cannot be determined from the data given.

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Quick Check ✓

- **Answer (B) is correct.** The coefficient of correlation (in standard notation, r) measures the strength of the linear relationship. The magnitude of r is independent of the scales of measurement of X and Y . Its range is -1.0 to 1.0 . A value of -1.0 indicates a perfectly inverse linear relationship between X and Y . A value of zero indicates no linear relationship between X and Y . A value of $+1.0$ indicates a perfectly direct relationship between X and Y . As X increases by 1, Y consistently decreases by 2. Hence, a perfectly inverse relationship exists, and r must be equal to -1.0 .

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Quick Check ✓

➔ **A regression equation**

- A. Estimates the dependent variables.
- B. Encompasses factors outside the relevant range.
- C. Is based on objective and constraint functions.
- D. Estimates the independent variable.

Quick Check ✓

- ➔ **Answer (A) is correct. Regression analysis is used to find an equation for the linear relationship among variables. The behavior of the dependent variable is explained in terms of one or more independent variables. Regression analysis is often used to estimate a dependent variable (such as cost) given a known independent variable (such as production).**

Learning Curves

Learning Curves

- **Learning curves** describe the fact that the more experience people have with something, the more efficient they become in doing that task. Higher costs per unit early in production are part of the start-up costs. It is commonly accepted that new products and production processes experience a period of low productivity followed by increased productivity.
- However, the rate of productivity improvement declines over time until it reaches a level where it remains, until another change in production occurs.

Two Learning-Curve Models

- There are two learning-curve models:
 - ✓ Cumulative Average-Time Learning Model
 - Evaluates the **average** time per unit required to produce a given number of units.
 - ✓ Incremental Unit-Time Learning Model
 - Evaluates the time needed to produce the **last unit** in a quantity of units.

Cumulative Average-Time Learning Model

- The Cumulative Average-Time Learning Model uses a constant percentage of decline in **average time per unit** each time that the cumulative quantity of units produced doubles.
- In other words, if a plant that manufactures automobiles is subject to an 80% learning curve, and if the time required to build the **first** automobile is 10 hours, then the total time required to manufacture **the first 2** autos will be 80% of (10 hours * 2), or 16 hours, which equates to an **average of 8 hours** for each automobile.

Cumulative Average-Time Learning Model (cont'd)

- Note that this model measures **total** time required, which includes the time for the first unit, and uses that total time to determine **average time per unit** for the entire amount produced. This is what “cumulative average” means.
- If learning had not taken place, it would have taken 20 hours ($2 * 10$ hours) to produce 2 autos. Thus, a learning rate of 1.00 or 100% is equivalent to no learning taking place.

Cumulative Average-Time Learning Model (cont'd)

- The maximum learning rate is 50%, or .50, because at a rate of 50%, the total time required for production of 2 automobiles would be equal to the time required to produce the first automobile ($100 * 2 * .5$), and it is impossible to produce 2 units in less time than it took to produce 1. Therefore, the learning rate will always be between 50% and 100%.

Example

- ➔ The cost accountant for Ray Lighting Manufacturing Company is planning production costs for a new lamp. Production of the new lamp will be subject to a 60% learning curve, since it involves only minimal adjustments to established processes. The initial lot of 500 lamps is expected to require 1,000 hours of labor.
- ➔ Costs are as follows:
 - ✓ Direct Labor \$20/hr.
 - ✓ Direct Materials \$50/DLH
 - ✓ Variable OH Applied \$25/DLH
 - ✓ Fixed OH Applied \$2,000/lot manufactured

Example (cont'd)

- ➔ **(1) What is the cumulative average time per unit after 8 lots have been manufactured, if the cumulative average-time model is used?**
- ➔ **Answer:** With a 60% learning curve, when the quantity of units produced doubles, the cumulative average time per unit for the doubled number of units is 60% of the cumulative average time per unit for the original number of units.

Example (cont'd)

- In this case, we are working with lots of 500 rather than units. However, the question asks for average time **per unit**, and there are 500 units in each lot.
- The first doubling will occur when the second lot of 500 has been produced. The second doubling will occur when the fourth lot of 500 has been produced. The third doubling will occur when the eighth lot of 500 has been produced.

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Example (cont'd)

- Therefore, the **total** number of labor hours required for 8 lots of 500 lamps is:
1000 hours for the first lot (500 units)
1000 hours X 2 (double) X60% =1200 (1000 units,2 lots)
1200 hours X 2 (double) X60% =1440 (2000 units,4 lots)
1440 hours X 2 (double) X 60%=1728 (4000 units ,8 lots)
- Or, $1,000 * (2 * .60) * (2 * .60) * (2 * .60) =$
- $1,000 * 1.2 * 1.2 * 1.2 = 1,728$ **total** labor hours required for 8 lots.

Example (cont'd)

- ➔ 1,728 total labor hours required ÷ 8 lots = average of 216 labor hours **per lot**.
- ➔ Average 216 labor hours per lot ÷ 500 lamps per lot = **.432** cumulative average number of labor hours required **per lamp** for 8 lots of 500 lamps each.

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Example (cont'd)

- ➔ **What is the total cost for the eighth lot?**
 - ✓ **Answer:** To determine an **incremental** cost under the cumulative average-time learning model, set up a chart such as the following:

Cum # of Lots	Cum Avg Time/Lot	Cum Tot Time	Cumulative Cost	Addition to Cum. Cost
1 *	1,000 =	1,000	$(\$95 * 1,000)^{(a)} + (\$2,000 * 1)^{(b)} = \$97,000$	\$97,000
2 *	600 =	1,200	$(\$95 * 1,200) + (\$2,000 * 2) = \$118,000$	21,000
4 *	360 =	1,440	$(\$95 * 1,440) + (\$2,000 * 4) = \$144,800$	26,800
8 *	216 =	1,728	$(\$95 * 1,728) + (\$2,000 * 8) = \$180,160$	35,360

Example (cont'd)

- Addition to Cumulative Cost is the cumulative cost of the total number of lots manufactured. The total number of lots manufactured is in the first column. Here, that is \$118,000 after the second lot. Subtract the previous cumulative cost, which was \$97,000 after the first lot, from it, and you will have the cost of only Lot 2.
- The Addition to Cumulative Cost for Lots 3 and 4 (the third doubling) is \$144,800 minus \$118,000, or \$26,800. Since there are 2 lots (Lots 3 and 4) that have cost a total of \$26,800, the cost of each lot is 1/2 that amount. Thus, Lot 3 costs \$13,400 and Lot 4 costs \$13,400.

Example (cont'd)

- The total cost for the **final 4 lots** (Lots 5, 6, 7 and 8) is \$35,360, which is \$180,160 minus \$144,160. Thus, the total cost for just the eighth lot is 1/4 of that, or \$8,840.
- (a) \$95 is the total variable cost per DLH: \$20 for direct labor, \$50 for direct materials, and \$25 for variable overhead applied. 1,000 is the number of hours required to manufacture the first lot. After 2 lots have been manufactured (the first doubling), the average time per lot will be 60% of 1,000 hours, or 600 hours, so the cumulative total time will be (600 hours * 2), or 1,200 hours.
- (b) \$2,000 is the fixed overhead applied per lot. Multiply the fixed overhead applied per lot by the number of lots (in column 1).

Incremental Unit-Time Learning Model

- The **Incremental Unit-Time Learning Model** states that the time needed to produce the **last unit** (incremental unit time) declines by a **constant percentage** each time the cumulative quantity of units produced doubles.
- In other words, using the same plant that is manufacturing automobiles, which is subject to an 80% learning curve rate, and the time required to build the **first** automobile is 10 hours, the total time required to manufacture **the second** auto will be 80% of 10 hours, or 8 hours. Thus, the total time required to produce 2 autos is 10 hours + 8 hours, or 18 hours. And the **average time** per unit will be $18 \div 2$, or 9 hours.

Incremental Unit-Time Learning Model (cont'd)

- Under the same assumptions for both models, the Incremental Unit-Time Model will predict a higher cumulative total time (and thus a higher average time per unit) to produce two or more units than will the Cumulative Average-Time Model. The choice of which model to use should be based on which more accurately predicts the behavior of labor hour usage as production levels increase.

Incremental Unit-Time Learning Model (cont'd)

- An example for the Incremental Unit-Time Learning Model is not included here. Calculations involving total time and average time per unit beyond the first doubling require the use of natural logarithms and either a financial calculator or a computer, and are thus beyond the scope of the exam.

Benefits of Learning Curve Analysis

- Decisions such as the following can be aided by learning curve analysis:
 - ✓ Life-Cycle costing – in calculating the cost of a contract, learning curve analysis can ensure that the cost estimates are accurate over the life of the contract, leading to better bidding.
 - ✓ Development of production plans and labor requirements – production and labor budgets should be adjusted to accommodate learning curves.

Limitations of Learning Curve Analysis

- ➔ There are three limitations and problems associated with learning curve analysis:
- ✓ Learning curve analysis is **appropriate only for labor-intensive operations involving repetitive tasks** where repeated trials improve performance. If the production process is designed to have fast set-up times using robotics and computer controls, there is little repetitive labor and thus little opportunity for learning to take place.
 - ✓ The **learning rate is assumed to be constant**. In real life, the decline in labor time might not be so constant. It might be that the learning rate would decline at the rate of 70% for the first 75,000 units, followed by 80% for the next 50,000 units and 95% for the next 25,000 units.

Limitations of Learning Curve Analysis (cont'd)

- ✓ The reliability of a learning curve calculation can be jeopardized because **an observed change in productivity might actually be associated with factors other than learning**. An increase in productivity might be due to a change in the labor mix, a change in the product mix or other factors. If that is the situation, a learning model developed using this historical data would produce inaccurate estimates of labor time and cost.

Example

- ➔ The average labor cost per unit for the first batch produced by a new process is \$120. The cumulative average labor cost after the second batch is \$72 per product. Using a batch size of 100 and assuming the learning curve continues, the total labor cost of four batches will be:
- A. \$4,320.
 - B. \$10,368.
 - C. \$2,592.
 - D. \$17,280.

Example (Cont'd)

- ➔ The first batch costs DL \$120 per unit (\$12000) & the second DL \$72 per unit , so the % of the learning curve $72/120 = 60\%$,

\$12000 for the first batch (100 units)

\$12000 X 2 (double) X 60% = \$14400 (200 units, 2 batches)

\$14400 X 2 (double) X 60% = \$17280 (400 units, 4 batches)

$120 \times .6 \times .6 \times 400 = \17280 or, $72 \times .6 \times 400 \text{ units} = \17280

Linear Programming

Scarce resource considerations
Capacity constraints

Linear Programming

- **Linear programming** is a method of problem solving that can aid in decision-making.
- Linear programming is used to either **maximize or minimize some quantity** (called the objective function).
- At the same time, this maximizing or minimizing must be accomplished in the presence of **constraints**, or **restrictions**, such as limited quantities of labor or materials. The maximization or minimization must be done without violating any of the constraints.

Linear Programming (Cont'd)

- Typically, linear programming is used to maximize the total contribution margin of a mix of products under the conditions of multiple constraints. However, the **objective function** may represent other goals as well. Profit levels, total revenues, total costs, pollution levels and percent return on an investment are a few examples.

Linear Programming (Cont'd)

- The set of **constraints** represents conditions that must be satisfied when determining levels for the decision variables. Constraints might be limited raw materials, equipment time, labor hours, or even limited demand for the products. The constraints in a linear programming problem are usually represented by **inequalities** (i.e., $<$ or \leq and $>$ or \geq types)¹

Note: The word "inequality" is used instead of "equation," because equation implies that both sides of an equation are equal. Constraint inequalities say that one side of the equation must be less than, less than or equal to, more than, or more than or equal to the other side. Thus, we cannot call these equations but we call them inequalities instead.

Linear Programming (Cont'd)

➤ Constraints in a linear programming model are of two types:

✓ **structural** constraints

➤ Structural constraints are used to express things such as resource limitations.

✓ **Nonnegativity** constraints.

➤ Nonnegativity constraints are included in order to make sure that decision variables will be only positive numbers. After all, we cannot manufacture -10 units of product!

Example

➤ Alpha Sports Company manufactures two different treadmills. One is its "Regular" model and the other is its "Hyper" model. Each model is processed through two departments: Molding and Finishing. The labor hour requirements in each department and the contribution margin information for each model are as follows. Also, the number of labor hours available per week in each department is shown.

	Regular (x)	Hyper (y)	Labor Capacity Per Week
Molding	4 hrs/unit	2 hrs/unit	600 hrs
Finishing	3 hrs/unit	5 hrs/unit	800 hrs
Selling price/unit	\$300	\$500	
Direct Material cost/unit	70	90	
Direct Labor cost/unit	100	235	
Contribution margin/unit	\$130	\$175	

Example (Cont'd)

- We want to determine the number of units to produce of each product to **maximize the total contribution margin** without exceeding the maximum labor hours available per week (our constrain).
 - ✓ It is called our **objective function**
 - MAX CM = \$130 x + \$175 y
- The first task is to determine the inequalities that express the constraints with respect to labor capacity. We will then find the solution set that satisfies all the inequalities.

Example (Cont'd)

- Letting x stand for the number of Regular treadmills produced in one week and y stand for the number of Hyper treadmills produced in one week
 - ✓ The first constraint is the number of labor hours available each week in the Molding Department :
$$4x + 2y \leq 600$$
 - ✓ The next constraint is the number of labor hours available each week in the Finishing Department :
$$3x + 5y \leq 800$$

Example (Cont'd)

It is important to recognize that we cannot produce a negative amount of either models. So, although it may seem obvious, we will add the two nonnegative constraints

$$x \geq 0$$

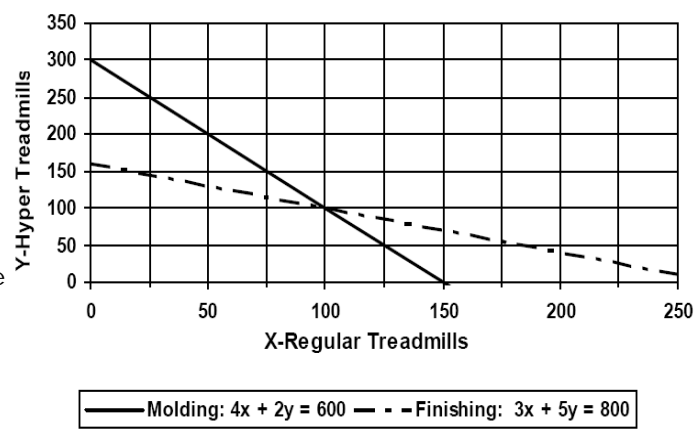
$$y \geq 0$$

- When working with inequalities, there can be an infinite number of values for x and y that will satisfy the inequalities.
- If we have a linear programming problem with only two variables, it can be solved graphically. The graph below illustrates the solution set for the above problem.

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Example (Cont'd)

Feasible solutions the area that is under both lines. All those combinations are called the **feasible solutions**. One of those feasible solutions will create the highest profit (i.e., contribution margin). Our mission is to find it.



Example (Cont'd)

➔ Here is the equation that expresses that objective. It is called the **objective function**.

✓
$$z = 130x + 175y$$

✓ Note that x & y are the controllable inputs (**decision variable**) in the problem .

✓ Where z represents the maximum contribution margin that can be attained for the mix of products, x represents the number of units of the Regular Treadmill, and y represents the number of units of the Hyper Treadmill to be manufactured each week.

➔ **The Corner Point Method: The optimal solution to a linear programming problem will be at one of the corner points in the area of feasible solutions.**

Example (Cont'd)

➔ Here are our four corner points on the graph above:

$x = 0, y = 160$

$x = 150, y = 0$

$x = 100, y = 100$

$x = 0, y = 0$

➔ We know profit will not be maximized if we produce nothing ($x = 0, y = 0$). Substituting

➔ the other three combinations into the objective function, we get:

$z = (130 * 0) + (175 * 160) = \$28,000$

$z = (130 * 150) + (175 * 0) = \$19,500$

$z = (130 * 100) + (175 * 100) = \$30,500$

➔ We find that profit will be maximized if we produce 100 Regular treadmills and 100 Hyper treadmills per week.

The Algebraic Method

- We can note that the optimal solution in this example is the point of the intersection of the two constraint lines, we can get the point of intersection by alternative method .
- An alternative method of solving a linear programming problem is the algebraic method. The algebraic method can be used to find the point of the intersection and it may be the optimal solutions for the majority of CMA questions.
- The point that result from the algebraic method is one of the feasible points not necessarily the optimal point.

The Algebraic Method (Cont'd)

- **Steps to solve the problem algebraically :**
 1. Change the constraint inequalities to equations;
 2. Solve the constraint equations simultaneously to find the unique solution;
 3. Then substitute the values you get for the variables into the objective function.
- This procedure to solve the problem **algebraically** called "Matrix Algebra" .

The Algebraic Method (Cont'd)

➤ Our constraint inequalities, expressed as equations, are:

$$4x + 2y = 600$$

$$3x + 5y = 800$$

➤ Express one of the equations in terms of x or y

Using $3x + 5y = 800$:

$$5y = 800 - 3x$$

$$y = 160 - .6x$$

The Algebraic Method (Cont'd)

➤ Substitute this value for y into either constraint equation to solve for x:

$$4x + 2y = 600$$

$$4x + 2(160 - .6x) = 600$$

$$4x + 320 - 1.2x = 600$$

$$2.8x + 320 = 600$$

$$2.8x = 280$$

$$x = 100$$

The Algebraic Method (Cont'd)

- Substitute the value you got for x above into the equation to solve for y:

$$4(100) + 2y = 600$$

$$400 + 2y = 600$$

$$2y = 200$$

$$y = 100$$

- Note that the answer, 100 Regular and 100 Hyper treadmills, is the same as the answer we got by graphing the problem, and in this example it is the optimal solution .

Example

- RCM ,Inc ., produce two product F , S. there are three raw materials blended for the production of the two finished products F,S.
- RCM production is constrained by a limited availability of the three raw materials.
- For the current production period , RMC has the following quantities of each raw material.

Material	Amount Available for production
M1	20 tons
M2	5 tons
M3	21 tons

Example (Cont'd)

➤ **Materials Requirements per ton :**

	Product	
	F	S
M1	0.4	0.5
M2		0.2
M3	0.6	0.3
CM	\$40	\$30

Example (Cont'd)

➤ **Write the Objective in terms of the Decision Variable.**

✓ Max CM = \$40F + \$30S,

➤ this objective subject to the following constraints (s,t).

➤ **Write the constraints in terms of the Decision Variable**

✓ M1 $0.4F + 0.5S \leq 20$

✓ M2 $0.2S \leq 5$

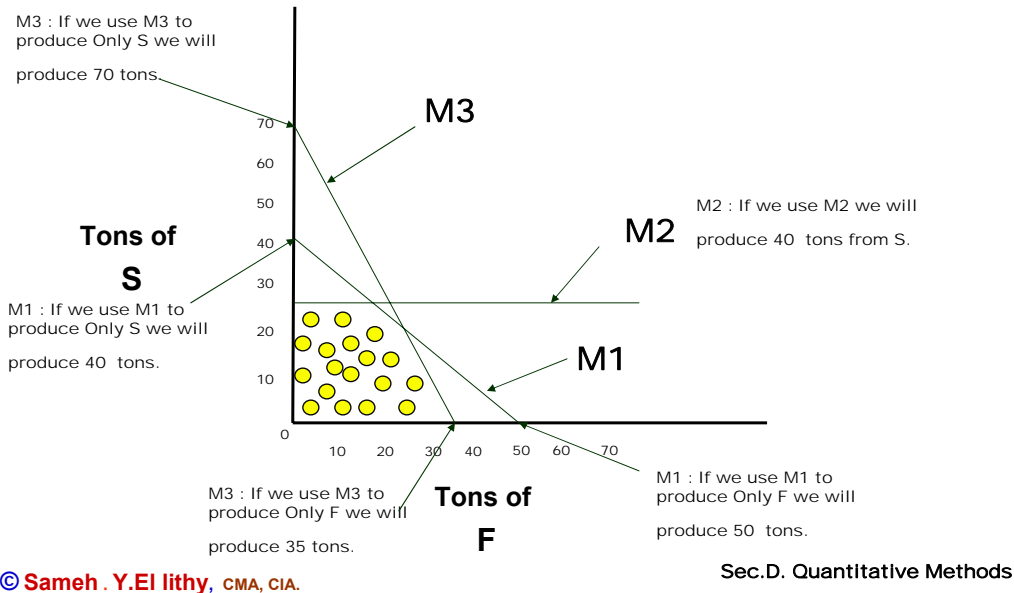
✓ M3 $0.6F + 0.3S \leq 21$

➤ **Add the Nonnegativity Constraint**

✓ $F \geq 0$ & $S \geq 0$ or $F, S \geq 0$

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LP Graphically



Example (Cont'd)

- We note that the optimal solution $F=25$, $S=20$ will require the full usage of M1 and M2, but only 4 tons of the five tons of M2.
- The 1 ton of unused material M2 (the unused capacity) called **Slack**.
- The optimal solution in RMC problem has the following values of the slack variables :

Constrain	Value of Slack Variable
M1	$S_1=0$
M2	$S_2=1$
M3	$S_3=0$

Example (Cont'd)

- By finding the optimal solution in the previous graph , we see that the material 1 constraint and the material 3 constraint restrict , or **bind**, the feasible region at this point.
- Thus , the optimal solution requires the use of all of these two resources. In other words , the graph shows that at the optimal solution material 1 and material 3 will have zero slack. But because the material 2 constraint is not binding the feasible region at the optimal solution , we can expect some slack for this resources.

Sensitivity Analysis & LP

- Sensitivity analysis is the study of how changes in the coefficients of a linear programming problem affect the optimal solutions .
- Using sensitivity analysis , we can answer the questions such as the following :
 - ✓ How will a change in an objective function coefficient affect the optimal solution?
 - ✓ How will a change in a right –hand-side value for a constraint affect the optimal solution?

POSTOPTIMALITY ANALYSIS

- Because sensitivity analysis is concerned with how these changes affect the optimal solution, sensitivity analysis does not begin until the optimal solution to the original linear programming problem has been obtained.
- For this reason, sensitivity analysis is often referred to as **postoptimality analysis**.

THE IMPORTANCE OF SENSITIVITY ANALYSIS

- Sensitivity analysis is important to decision makers because real world problems exist in a changing environment. Prices of raw material change, product demand change, and so on. If a linear programming model has been used in such an environment, we can expect some of the coefficients in the model to change over time.
 - ✓ e.g., economic conditions, is a type of the decision variables outside the control of the decision maker, so it called "Exogenous variables".

THE IMPORTANCE OF SENSITIVITY ANALYSIS (Cont'd)

- ➔ As a result , we will want to determine how changes affect the optimal solution. Sensitivity analysis provides information needed to respond to such changes without requiring a complete solution of a revised linear programming.

APPLICATION OF SENSITIVITY ANALYSIS TO OUR PROBLEM

- ➔ The optimal solution, $F = 25$ tons, and $S = 20$ tons , provided maximum $CM = \$1600$.
- ➔ Using sensitivity analysis , we can answer the question such as the following two question :
 - ✓ How will a change in an objective function coefficient affect the optimal solution?
 - ➔ What-If the CM of F fall from $\$40$ to $\$30$, is the original solution (produce 25 tons F , 20 tons S) still the optimal solution?

APPLICATION OF SENSITIVITY ANALYSIS TO OUR PROBLEM (Cont'd)

- ✓ How will a change in a right –hand-side value for a constraint affect the optimal solution?
 - What-If RMC could obtain additional quantities of either of the resources?
 - Note that the values in the right hand side of the constraint called “Shadow Prices” in the Excel application for LP.

More in Shadow Prices

- A **shadow price** is the amount by which the value of the optimal solution of the objective function in a linear programming problem will change if a one-unit change is made in a binding constraint.
 - ✓ A nonbinding constraint is one that has excess capacity; i.e., the optimal solution does not use all of the given resource. The shadow price for a nonbinding constraint is zero because a one-unit change will not affect the optimal solution when excess capacity exists.

More in Shadow Prices (Cont'd)

- The shadow cost is the amount by which the objective would increase with an increase in the inputs. This amount that is given up by not having one more unit of input is a type of opportunity cost.
 - ✓ Recall that :.An opportunity cost is the lost benefit from not using a resource for its next best alternative use.
- The calculation of shadow prices is a simple example of **sensitivity analysis**, which is any procedure to test the responsiveness of the solution indicated by a model to changes in variables, alternative decisions, or errors.

Quick Check ✓

- In linear programming, the shadow price refers to the
 - A. Measurement of the value of relaxing a constraint in a problem with dual variables.
 - B. Marginal change in profit associated with a change in the contribution margin of one of the variables.
 - C. Unused capacity available once the optimal solution is obtained.
 - D. Variables that are included in the final solution of the linear programming model.

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Quick Check ✓

- Answer (A) is correct. A shadow price is the amount by which the value of the optimal solution of the objective function will change if a one-unit change is made in a binding constraint. The calculation of shadow prices is an example of sensitivity analysis, which is any procedure that tests the responsiveness of a solution to changes in variables

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Quick Check ✓

- Sensitivity analysis in linear programming is used to
- A. Determine the degree that the constraints vary.
 - B. Test the accuracy of the parameters.
 - C. Develop the technological matrix.
 - D. Determine how the optimal solution would react to changes in parameters.

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Quick Check ✓

- ➔ Answer (D) is correct. Sensitivity analysis in linear programming determines how the optimal solution will change if an objective function coefficient, the limiting value of a resource constraint, or a constraint coefficient is varied. It also considers the effect of adding a new variable or constraint.

TYPICAL APPLICATIONS OF LINEAR PROGRAMMING

- ➔ A marketing manager wants to determine how best to allocate a fixed advertising budget among alternative advertising media such as radio , television , and magazines . The manager would like to determine the media mix that **maximizes** advertising effectiveness .
- ➔ A financial analyst must select an investment portfolio from a variety of stock and bond investment alternatives . The analyst would like to establish the portfolio that **maximizes** the return on investment.

TYPICAL APPLICATIONS OF LINEAR PROGRAMMING (Cont'd)

- A company has warehouses in a number of locations . Given specific customer demands , the company would like to determine how much each warehouse should ship to each customer so that total transportation costs are *minimized*. This application has a special importance in the text books under what is known as Transportation Model as a special type of LP problem.

TYPICAL APPLICATIONS OF LINEAR PROGRAMMING (Cont'd)

- A manufacturer should *minimize* production costs while satisfying production requirements, maintaining required inventory levels, staying within production capacities, and using available employees. The objective function is the production cost to be minimized; the constraints are production requirements, inventory levels, production capacity, and available employees.

TYPICAL APPLICATIONS OF LINEAR PROGRAMMING (Cont'd)

- Other business applications include
 - ✓ Selecting a product mix
 - ✓ Blending chemical products
 - ✓ Scheduling flight crews
 - ✓ Assigning jobs to machines
 - ✓ Determining transportation routes

Quick Check ✓

- A large retail chain has 10 warehouses and 50 retail locations. The quantitative technique it would likely use to determine the minimum cost of moving its goods from the warehouses to the retail stores is
 - A. A transportation model.
 - B. Markov analysis.
 - C. Calculus-based optimization.
 - D. A queuing model.

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Quick Check ✓

- Answer (A) is correct. The transportation model is a special application of linear programming to a type of network flow problem. It determines the minimum cost of, or maximum profit or revenue derived from, physically moving goods or delivering services from sources of supply (warehouses) to other destinations (retail stores).

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Quick Check ✓

- The best way to allocate scarce resources to attain a chosen objective such as maximization of operating income or minimization of operating costs is
- A. Relevant costing.
 - B. Responsibility accounting.
 - C. Simple regression analysis.
 - D. Linear programming.

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Quick Check ✓

- ➔ Answer (D) is correct. Linear programming is a deterministic mathematical technique used to maximize linear revenue functions or to minimize linear cost functions subject to linear constraints. Linear programming is often used to plan resource allocations. Managers need to use resources as profitably or as inexpensively as possible. Solving linear programming problems requires the use of independent variables, dependent variables, constraints, and slack variables. Slack variables are used to convert equations stated as inequalities into equalities.

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Quick Check ✓

- ➔ Lynch Corporation manufactures and sells three products. Top management would like to change the present sales mix to increase overall operating income. Each of the company's products has various manufacturing and sales restrictions, including maximum demand levels and raw material availability. In order to achieve the most profitable sales mix, Lynch should employ
- ✓ Multiple regression.
 - ✓ Linear programming.
 - ✓ Mix variance analysis.
 - ✓ Expected value techniques.

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Quick Check ✓

- ➔ Answer (B) is correct. Linear programming is used to maximize an objective function, such as contribution margin, given a number of constraints. The constraints represent limits on sales (product demand) and limitations on the supply of resources, such as raw materials, labor, and machine hours.

LIMITATIONS OF LP

- ➔ The biggest drawback to linear programming is that the decision problem must be expressed in linear functions.
- ✓ For example, this requires the assumption that all costs are either variable or fixed costs. If the relationships involved are not linear, then linear programming will not provide an answer. Thus linear programming technique is effective only for **Straight-line relationship situations**.
- ➔ The linear programming model is designed to maximize income or minimize costs given resource constraints. It is a **static** model because it does not allow for a sequence of decisions over time .

Goal Programming Vs. Linear Programming

- **Goal programming permits an ordinal specification and solution to a problem with multiple goals.**
 - ✓ With its ability to incorporate multiple goals (financial & nonfinancial),
 - ✓ Goal programming has the potential to be applied to problems with financial, social, and environmental objectives. It was first introduced as a means of solving linear programming problems that did not have a feasible solution.
- **linear programming cannot deal with multiple objectives.**

Quick Check ✓

- Keego Enterprises manufactures two products, boat wax and car wax, in two departments, the Mixing Department and the Packaging Department. The Mixing Department has 800 hours per month available, and the Packaging Department has 1,200 hours per month available. Production of the two products cannot exceed 36,000 pounds. Data on the two products follow:

	Contribution Margin (per 100 pounds)	Hours per 100 Pounds	
		Mixing (M)	Packaging (P)
Boat wax (B)	\$200	5.0	3.6
Car wax (C)	150	2.4	6.0

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Quick Check ✓

- The objective function for the linear program Keego would use to determine the optimal monthly production of each wax would be:
- A. $Z = 150B + 200C$.
 - B. $2B + 1.5C \geq 36,000$.
 - C. $2B + 1.5C \leq 36,000$.
 - D. $Z = 200B + 150C$.

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Quick Check ✓

- Answer (D) is correct. The objective function is the equation to maximize the contribution margin. Given that each 100 pounds of boat wax contributes \$200 and each 100 pounds of car wax contributes \$150, the company's goal is to maximize the total of these amounts. Consequently, if B is the number of 100-pound units of boat wax, C is the number of 100-pound units of car wax, and Z is the total contribution margin, the objective function is $Z = \$200B + \$150C$.

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Quick Check ✓

- The mixing constraint for the Keego linear program would be:
- A. $2.4M + 6P \geq 36,000$.
 - B. $5B + 2.4C \geq 800$.
 - C. $5B + 2.4C \leq 800$.
 - D. $5B + 2.4C = 800$.

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Quick Check ✓

- Answer (C) is correct. The Mixing Department has only 800 hours available per month, which is a limitation (constraint) on production. Thus, the total mixing time is less than or equal to 800 hours. Every 100-pound batch of boat wax (B) requires 5 hours of mixing time, and every 100-pound batch of car wax (C) requires 2.4 hours of mixing time. Accordingly, the total mixing time available cannot exceed 5B plus 2.4C, and the constraint equation is $5B + 2.4C \leq 800$.

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H.W

- ➔ The Multi Resource Company manufactures two lines of washing machines, Regular and Deluxe. The contribution margin of a Regular model is \$110 and a Deluxe model \$175. The company has two departments, assembly and testing. The Regular model requires 3 hours to assemble, while a Deluxe model requires 4 hours. The total time available in assembly is 12,000 hours. In the testing department, it requires 2.5 hours to test a Regular model and 1.5 hours to test a Deluxe model. A total of 6,000 hours of testing time is available. To maximize profit, Multi Resource should manufacture ??

Project Scheduling – Network Analysis

- Critical Path Method (CPM)
- Program Evaluation Review
Technique (PERT)

Project Scheduling – Network Analysis

- Project scheduling is the process of planning, managing and controlling large projects that are composed of many different jobs performed by many different departments and people.
- When projects are very large and complex, the manager needs a system for keeping track of all the information and coordinating the various activities in order to complete the entire project on time.

Project Scheduling – Network Analysis

(Cont'd)

- Many activities in a project are dependent upon the completion of other activities, and they cannot begin until the other activities have been completed. Some activities are critical because they must be completed exactly as scheduled to avoid slowing down the whole project, whereas other activities are noncritical and may be delayed for a time before they will cause a slowing of the entire project.

Gantt charts or bar charts

- Henry L. Gantt developed the Gantt Chart as a graphical aid to scheduling jobs on machines in 1918.
- This application was the first of what has become known as project scheduling techniques.
- **Gantt charts** or **bar charts** are simple to construct and use. To develop a Gantt chart, divide the project into logical subprojects called activities or tasks. Estimate the start and completion times for each activity. Prepare a bar chart showing each activity as a horizontal bar along a time scale.

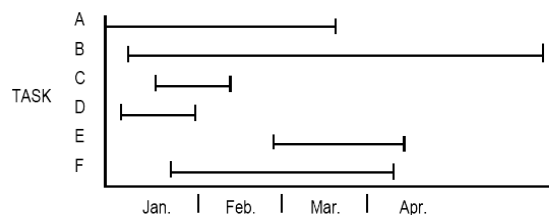
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Gantt charts or bar charts

- The major advantage of the Gantt chart is its simplicity. It forces the planner to think ahead and define logical activities. As the project progresses, actual completion times can be compared with planned times. Furthermore, the technique requires no special tools or mathematics and can be used on small projects as well as large ones.
- The major disadvantage is that interrelationships among activities are not shown. Several special methods have been developed to show these on a Gantt chart, but they are feasible only for simple relationships.



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PERT/CPM

- When we faced with large and complex projects we have to use methods complicated than Gantt Charts.
- Two widely used project scheduling techniques are:
 - ✓ Program Evaluation and Review Technique (PERT), and
 - ✓ Critical Path Method (CPM)

Development of PERT and CPM

- PERT and CPM have the same general purpose, in that they both address the same issues. The two techniques were developed independently in the late 1950s.
- PERT was developed by the U.S. Navy primarily to handle projects where the time required for each activity was uncertain.
- CPM was developed for use in industrial projects where the time requirements for each activity were known. CPM introduces the concept of trade-offs between time and cost for the various project activities.

Development of PERT and CPM

- Computer applications for PERT and CPM have combined the two approaches, using the best features of both. Therefore, a distinction between the two techniques is no longer needed, and they are referred to as PERT/CPM.

PROJECT SCHEDULING WITH KNOWN ACTIVITY TIMES

- The most important concept of PERT/CPM is that one group of activities controls the entire project, because it is the set of activities that will take the longest time to complete. Thus, management resources should be concentrated on these “critical” activities, which will determine the fate of the entire project. Other, less critical, activities can be rescheduled if necessary and resources for them can be reallocated without affecting the whole project.

PROJECT SCHEDULING WITH KNOWN
ACTIVITY TIMES (Cont'd)

- ➔ PERT/CPM involves graphical representations of the project, called the **project network**. The project's beginning, end and each activity are represented by **nodes** on the network. Lines, or **arcs**, connect the nodes and show the relationships between and among them. The project network helps the manager visualize the activity relationships and assists in carrying out the PERT/CPM computations.

PROJECT SCHEDULING WITH KNOWN
ACTIVITY TIMES (Cont'd)

- ➔ Once we have the form of the project network, we can estimate the time required by each activity, the set of critical activities, and the time required for the whole project. Each activity, represented by a node, is assigned a time that will be required for its completion.

PROJECT SCHEDULING WITH KNOWN ACTIVITY TIMES (Cont'd)

- After acquiring the expected times for each activity, we can determine which path is the critical path. A **path** through the network is a series of connected nodes that go all the way from the beginning to the end of the project. A network may have many paths, and all the paths must be completed in order to complete the project. The **critical path** is the path that requires the most time because if activities on that path are delayed for any reason, the entire project will be delayed. Activities on the critical path are called **critical activities** for the project.

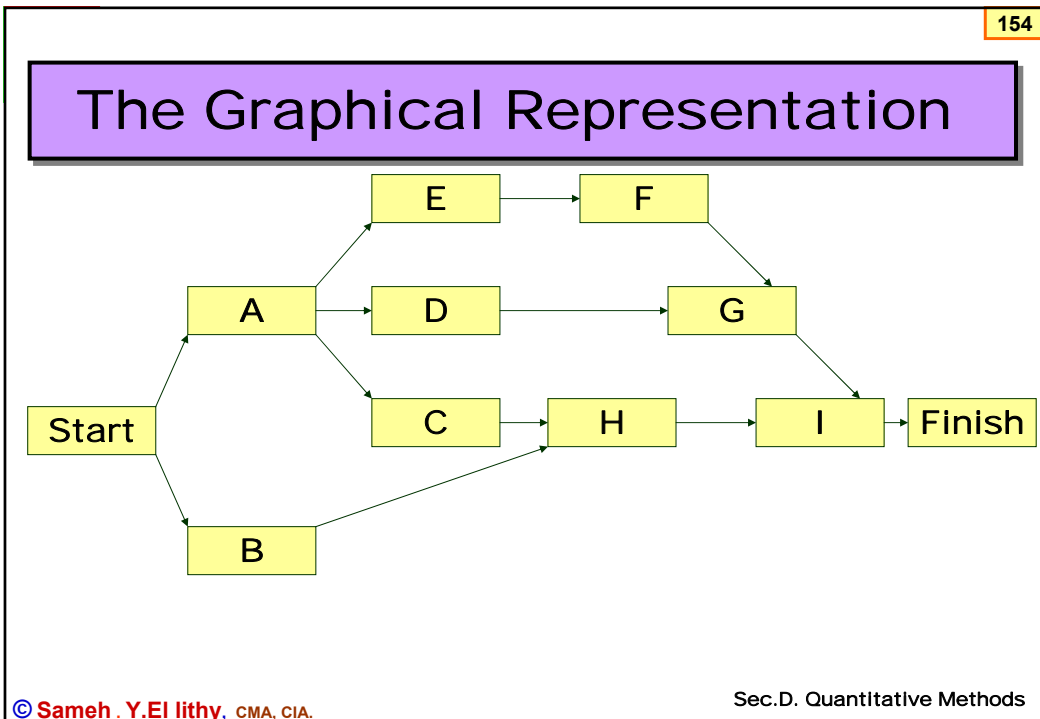
EXAMPLE

- The owner of the Western hills shopping center is planning to modernize and expand the current 32-business shopping center complex.
- This project with nine activities denoted **A** through **I**. The next table shows the nine activities and also shows the **immediate predecessor** for a given activity. That is , the activities that must be completed **immediately** prior to the start of that activity.

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List of the Activities for the Project		
Activity	Immediate predecessor	Activity time
A	—	5
B	—	6
C	A	4
D	A	3
E	A	1
F	E	4
G	D,F	14
H	B,C	12
I	G,H	2

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THE CRITICAL PATH

➤ The **critical path** is the path that requires the most time because if activities on that path are delayed for any reason, the entire project will be delayed. Activities on the critical path are called **critical activities** for the project. We have to identify the critical path , but before doing so , we need to identify the possible paths in the network, we will find four possible paths ,

- ✓ A-E-F-G-I = $5+1+4+14+2=26$
- ✓ A-D-G-I = $5+3+14+2=24$
- ✓ A-C-H-I = $5+4+12+2=23$
- ✓ B-H-I = $6+12+2=20$

THE CRITICAL PATH

➤ All the four possible paths must be traversed in order to complete the project , so we will look for the path that requires the most time. Because all other paths are shorter in duration , this **longest** path determines the total time required to complete the project. If activities on the path are delayed , the entire project will be delayed. Thus the longest path is the **critical path**.

Start Times, Finish Times, Slack Times and the Critical Path

- An important part of determining the critical path is determining start times, finish times and slack time **for each individual activity** in a project. This appears difficult, but it is really just common sense. It is beneficial to put in the effort to understand how it works.

Start Times, Finish Times, Slack Times and the Critical Path (Cont'd)

- In determining start times, we need to know the earliest and latest possible start times for each activity. We determine the earliest start time by counting from the left side of the diagram. We determine the latest start time by counting from the right side.
- For finish times, we need the same thing: the **earliest and latest possible finish times** for each activity. Once these are known, we will know where there is slack time and we will know the critical path, or paths (there can be more than one critical path).

Start Times & Finish Times

Activity Name	Earliest Start Time (ES)	Earliest Finish Time (EF)
A		
Activity Time (t)	Latest Start Time (LS)	Latest finish Time (LF)

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Earliest Time (Forward pass)

- $EF = ES + t$
 - ✓ Activity A can start as soon as the project starts , so we can set the earliest start time for activity A equal to zero. With an activity time of 5 weeks , the earliest finish time for activity A is $EF = ES + t = 0 + 5 = 5$.
- Because an activity cannot be started until *all* immediately preceding activities have been finished , the following rule can be used to determine the ES for each activity .
- The ES for an activity is equal to the **largest** of the EF for all its immediate predecessors.

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Latest Time (Backward pass)

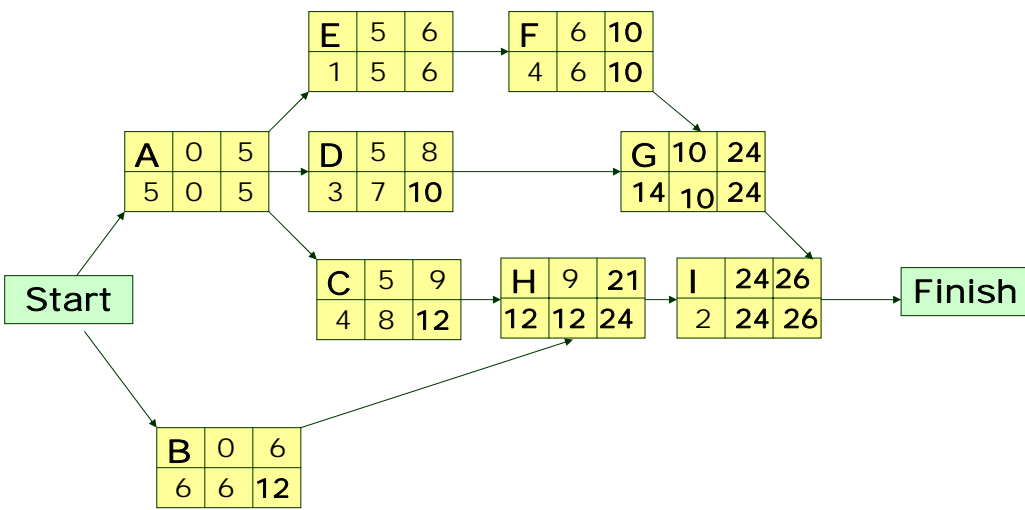
➔ $LS = LF - t$

✓ Since the project can be completed in 26 weeks, we begin the backward pass with the latest finish time of 26 for activity I. Once the latest finish time for an activity is known, the latest start time for an activity can be computed as $LS = LF - t$.

- ➔ The following rule can be used to determine the latest finish time for each activity in the network.
- ➔ The LF for an activity is the **smallest** of the LS for all activities that immediately follow the activity.

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Earliest Time & Latest Time



SLACK TIME

- After we have completed the forward and backward passes , we can determine the amount of slack associated with each activity.
- Slack is the length of time an activity can be delayed without increasing the project time . The amount of slack for an activity is computed as follows :
 - ✓ Slack = LS – ES = LF - EF

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SLACK TIME (Cont'd)

Activity	Slack	Critical Path?
A	0	Yes
B	6	---
C	3	---
D	2	---
E	0	Yes
F	0	Yes
G	0	Yes
H	3	---
I	0	Yes

PROJECT SCHEDULING WITH UNCERTAIN ACTIVITY TIMES

Probabilistic Technique Originally PERT

PROJECT SCHEDULING WITH UNCERTAIN ACTIVITY TIMES

- If a project has been done before, such as building a house using a standard floor plan and standard materials, the construction manager should be able to make accurate estimates of the time required for each activity. However, if the house is a custom home that requires unfamiliar materials to the builder, activity times may be uncertain.
- When activity times are uncertain, they are best described by a **range** of possible times rather than one single time estimate. These uncertain activity times are treated as random variables with probability distributions.

PROJECT SCHEDULING WITH UNCERTAIN ACTIVITY TIMES (Cont'd)

- ➔ To determine an **expected time** for an activity when its time is uncertain, we need three time estimates: its **optimistic time** (a), **most probable time** (m), and **pessimistic time** (b). To calculate the **expected completion time** for an individual activity using these three time estimates, we use the following formula:

$$\frac{\text{Pessimistic time } (a) + (\text{Most Likely time } (m) * 4) + \text{Optimistic time } (b)}{6}$$

Quick Check ✓

- ➔ A company is planning a multi-phase construction project. The time estimates for a particular phase of the project are
- | | |
|-------------|----------|
| Optimistic | 2 months |
| Most likely | 4 months |
| Pessimistic | 9 months |
- ➔ Using the Program Evaluation Review Technique (PERT), the expected completion time for this particular phase is
- 2 months.
 - 4 months.
 - 4.5 months.
 - 5 months.

Quick Check ✓

- Answer (C) is correct. The PERT method weights expected completion times using a 1-4-1 ratio. The most optimistic and pessimistic estimates are weighted equally, but the most likely completion time is weighted four times more heavily than the others. Thus, the PERT estimate is 4.5 months $\{2 + (4 \times 4) + 9\} \div 6$.

PROJECT SCHEDULING WITH UNCERTAIN ACTIVITY TIMES (Cont'd)

- Network planning using three time estimates for each activity is called a **probabilistic** technique this used through PERT, because it allows for uncertainty.
- ✓ Probabilistic model : A model in which the uncontrollable inputs are uncertain and subject to variation . This is in contrast to deterministic techniques, or techniques that use only one time estimate for each activity (the uncontrollable inputs are known and cannot vary) .
- Large differences between the pessimistic and the optimistic times indicate a high degree of uncertainty about the time required for an activity , we can determine the standard deviation (σ) of an individual activity as follows:

$$\sigma = \frac{\text{Pessimistic time} - \text{Optimistic time}}{6}$$

PROJECT SCHEDULING WITH UNCERTAIN ACTIVITY TIMES (Cont'd)

- **With uncertain activity time we can use the variance to describe the dispersion or variation in the activity time values.** The **variance** of an individual activity is the square of the standard deviation, or σ^2 .
 - ✓ Expected time & variances & standard deviation all collected in a schedule called Beta schedule.
- When activity times are uncertain, the manager must remember that the calculation of the critical path will determine only the **expected time** to complete the project. The **actual** time required to complete the project may be quite different. Activities with larger variances have a greater degree of uncertainty.

PROJECT SCHEDULING WITH UNCERTAIN ACTIVITY TIMES (Cont'd)

- Therefore, the progress of any activity with a large variance should be closely monitored even if, based on its expected time, the activity does not appear to be a critical activity on the critical path.
- The **standard deviation of the completion time of the critical path** is calculated by taking the square root of the sum of the variances of all of the individual activities in the path. Remember that the variance of an activity is the square of the standard deviation of the activity.

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Quick Check ✓

- In a PERT network, the optimistic time for a particular activity is 9 weeks, and the pessimistic time is 21 weeks. Which one of the following is the best estimate of the standard deviation for the activity?
- A. 2
 - B. 6
 - C. 9
 - D. 12

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Quick Check ✓

- Answer (A) is correct. PERT analysis includes probabilistic estimates of activity completion times. Three time estimates are made: optimistic, most likely, and pessimistic. The time estimates for an activity are assumed to approximate a beta probability distribution. Accordingly, if the pessimistic and optimistic times are 21 and 9 weeks, respectively, the standard deviation is 2 weeks ($12 \div 6$).

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Cost-Time Tradeoffs and "Crashing"

Originally CPM

Cost-Time Tradeoffs and "Crashing"

- When CPM was originally developed, it was used not only for scheduling, but it was also used to determine what activities could be shortened by adding resources, thus shortening the completion time for the whole project.
- It also considered what the added cost of those resources needed to shorten the activity would be. This **cost-time tradeoff** enabled the manager to determine whether the additional cost involved in shortening the time to complete the project would be worthwhile.

Cost-Time Tradeoffs and "Crashing"

(Cont'd)

- If a project needs to be completed in less time, the critical path must be shortened. This can be done either by
 - ✓ using the existing resources in the company in a different manner (moving them from jobs with slack time to the critical path) or
 - ✓ bringing in additional resources.
 - Which choice a company makes will depend upon the skills of the resources in the company and whether or not they will be able to perform the needed tasks in the critical path.

Cost-Time Tradeoffs and "Crashing"

(Cont'd)

- Putting additional resources to work on specific activities to shorten the time to complete a project is called **crashing**. In order to decide where to crash, we need to know the least amount of crashing that is needed to get the project completed within the timeframe. We then determine what activities will cost the least to crash per unit of shortened project time.
- **The activities on the critical path are prime candidates for crashing.** However, if we crash those activities too much, then they might be shortened too much, making another path critical and wasting some of the additional resources for which we are paying extra. So the entire network needs to be examined and the crashing needs to be carefully planned.

Benefits of PERT/CPM

1. It forces managers to plan projects in intricate detail,
2. It can be used for scheduling,
3. It can be used to assign existing resources to a project in the most effective manner, and
4. It can be used to calculate costs to shorten the time required for a project.
5. Show the critical path through a network (Gantt chart does not).

Limitations and Criticisms of PERT/CPM

1. PERT is **extremely complicated**, and when costs are included in the analysis, scheduling complexity is increased. Furthermore, CPM does not deal with the influence of indirect costs and contractual incentives. It assumes that time and costs are linearly related, which they may not be.
2. It can lead to **overly optimistic estimates**. It can be misleading to look only at the critical path because paths that are near-critical and that have large variances may become critical. PERT/CPM does not account for these activities.

Quick Check ✓

- California Building Corporation uses the Critical Path Method to monitor construction jobs. The company is currently 2 weeks behind schedule on Job #181, which is subject to a \$10,500-per-week completion penalty. Path A-B-C-F-G-H-I has a normal completion time of 20 weeks, and critical path A-D-E-F-G-H-I has a normal completion time of 22 weeks. The following activities can be crashed.

Activities	Cost to Crash	
	1 Week	2 Weeks
BC	\$8,000	\$15,000
DE	10,000	19,600
EF	8,800	19,500

Quick Check ✓

- California Building desires to reduce the normal completion time of Job #181 and, at the same time, report the highest possible income for the year. California Building should crash:

- A. Activity BC 1 week and activity EF 1 week.
- B. Activity EF 2 weeks.
- C. Activity DE 1 week and activity EF 1 week.
- D. Activity DE 2 weeks.

Quick Check ✓

- ➔ Answer (c) is correct. Activities that are to be crashed in a critical path problem should be ones that are on the critical (longest) path. Thus, activity BC should not be selected because it is not on the critical path. Therefore, the only feasible choices are DE and EF on the critical path. The total cost to crash DE and EF for 1 week each is \$18,800 (\$10,000 + \$8,800), which is less than the cost to crash either activity for 2 weeks. Thus, DE and EF should be crashed for 1 week each since the total cost is less than the \$21,000 (\$10,500 x 2) 2-week delay penalty.

Quick Check ✓

- ➔ PERT and the critical path method (CPM) are used for
- A. Determining the optimal product mix.
 - B. Project planning and control.
 - C. Determining product costs.
 - D. Determining the number of servers needed in a fast food restaurant.

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Quick Check ✓

- Answer (B) is correct. The Program Evaluation and Review Technique and critical path method are useful in the planning and control of a complex system or process. PERT and CPM both construct a network of time relationships between each event or subproject to identify the subprojects that have a direct effect on the completion date of the project as a whole. The critical path is the longest path through the network. If any activity on this path takes longer than expected, the entire project will be delayed (slack time is zero). Every project has at least one critical path. Some have more than one.

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Quick Check ✓

- When making a cost/time trade-off in PERT analysis, the first activity that should be crashed is the activity
- A. With the least amount of slack.
 - B. On the critical path with the lowest unit crash cost.
 - C. On the critical path with the maximum possible time reduction.
 - D. With the lowest unit crash cost.

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Quick Check ✓

- Answer (B) is correct. When making a cost/time trade-off, the first activity to be crashed (have its completion time accelerated) is one on the critical path. To select an activity on another path would not reduce the total time of completion. The activity chosen should be that whose completion time can be accelerated at the lowest possible cost per unit of time saved.

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Quick Check ✓

- In the Program Evaluation Review Technique (PERT), slack is the
- A. Uncertainty associated with time estimates.
 - B. Difference between the latest starting time and earliest finishing time.
 - C. Path that has the largest amount of time associated with it.
 - D. Number of days an activity can be delayed without forcing a delay for the entire project.

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Quick Check ✓

- Answer (D) is correct. PERT diagrams are free-form networks showing each activity in a large project as a line between events. The critical path is the longest path in time through the network. That path is critical in that if any activity on the critical path takes longer than expected, the entire project will be delayed. Paths that are not critical have slack time. Slack is the number of days an activity can be delayed without forcing a delay for the entire project.

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Quick Check ✓

- The critical path through a network is the
- A. Shortest path.
 - B. Path with the most nodes.
 - C. Longest path.
 - D. Path with the most slack.

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Quick Check ✓

- ➔ Answer (C) is correct. The critical path is the longest path through the network. If any activity on this path takes longer than expected, the entire project will be delayed (slack time is zero). Every project has at least one critical path. Some have more than one.

Probability Concepts

Probability & Expected value

Probability

- There are many events in business and nature for which it is impossible to exactly predict the outcome. However, the occurrence of such events can be described quantitatively (numerically) if they occur a great number of times in finite conditions.
- For example, it is impossible to predict with 100% certainty if a fair coin will turn up as “heads” or “tails” in one particular toss of the coin. But if we toss that coin many times, we can conclude that the number of “heads” will be very close to 50% of the total number of flips of the coin.

Probability (Cont'd)

- **Probability** gives us a means of measuring numerically how likely it is that an event will occur. This is important in decision-making because it enables us to quantify and analyze uncertainties.
- Probability is always expressed as a value between 0 and 1.
 - ✓ The closer the probability is to 0, the less likely it is that the event will occur; a probability of 0 means there is no chance that the event will occur.
 - ✓ Conversely, a probability near 1 tells us that the event is almost certain to occur, and a probability of exactly 1 would mean it is absolutely certain to occur.

Two Requirements of Probability

- When the weather forecaster says there is a 40% probability that it will rain today, it also implicitly means there is a 60% probability that it **won't** rain. This illustrates the **two basic requirements of probability**:
1. The probability values assigned to **each** of the possible outcomes must be between 0 and 1; AND
 2. The probable values assigned to **all** of the possible outcomes must total 1.

Expected Value

- The **expected value** of a discrete random variable is the **weighted average of all the possible values** of the random variable. The weights are the probabilities for each of the values.
- The expected value is the **mean** value, also known as the **average** value.
- The symbol for the mean, average or expected value is μ , which is the Greek letter pronounced "mu."
- We will construct a comprehensive example illustrating all the tested terms .

Risk, Uncertainty and Expected Value

- There are many definitions of **risk**. One definition is “a condition in which there is a possibility of an adverse deviation from a desired outcome.” This is risk defined in its negative connotation.
- However, in a very real sense, risk does not carry a negative connotation. Where investments are concerned (both capital investments and security investments), risk is the possibility that an investment’s **actual** return will differ from its **expected** return. This difference may be either a positive difference or a negative difference.

Risk, Uncertainty and Expected Value

(Cont'd)

- **Risk** for a security can be measured by the **variability** of its historical returns or the **dispersion** of its historical returns around their average, or mean, return. Thus, risk is measured by **variance** and **standard deviation**.
- **Uncertainty** is risk that cannot be measured. In discussing a capital investment, we may or may not have information about historical returns on similar investments. If there is no information about historical returns for a particular investment, we are in the position of **decision-making under a condition of uncertainty**. When we are in this position, the probability distribution of possible returns must be determined **subjectively**.

Decision Analysis

Decision Making with & without Probability

Comprehensive Example

- PDC has purchased land , which will be the site of a new luxury condominium complex . PDC plans to price the individual condominium units between \$300,000 and \$1,400,000 . PDC has preliminary design with three different size of the condominium .
 - ✓ One with 30 cond.
 - ✓ One with 60 cond.
 - ✓ One with 90 cond.
- The financial success of the project depend upon the size of the cond. Complex and the chance event concerning the demand for the cond.
- The statement of the PDC decision problem is to select the size of the new cond. Project that will lead to the largest profit giving the uncertainty concerning the demand for cond..

Comprehensive Example (Cont'd)

- So , it is clear that the decision is to select the best size for the cond. complex .
- PDC has the following three decision alternatives
 - ✓ d₁ = a small complex with 30 cond.
 - ✓ d₂ = a medium complex with 60 cond.
 - ✓ d₃ = a large complex with 90 cond.
- A factor in selecting the best decision alternatives is the uncertainty associated with the chance event concerning the demand for the cond.

Comprehensive Example (Cont'd)

- When asked about the possible demand for the cond. , the PDC's president acknowledge a wide range of possibilities , but decided that it would be adequate to consider two possible **chance event outcomes** :
 - ✓ Strong demand and ,
 - ✓ Weak demand.
- In decision analysis the possible outcomes for a chance event are referred to as **states of nature** . The state of nature are defined so that one and only one of the possible states of nature will occur .

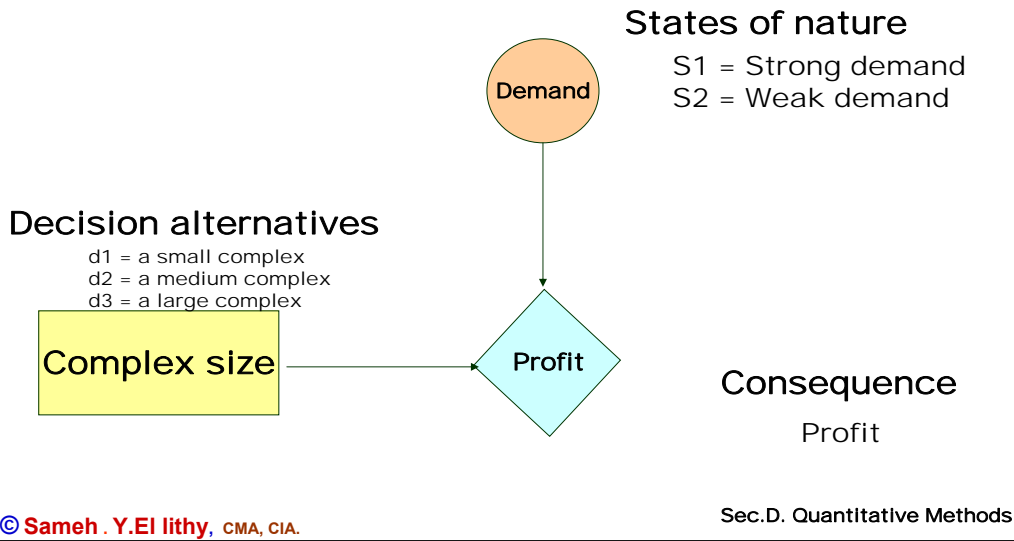
Comprehensive Example (Cont'd)

- For the PDC problem , the chance event concerning the demand for conds. has two states of nature :
 - ✓ S1 = Strong demand for the conds.
 - ✓ S2 = Weak demand for the conds.
- Thus , management must first select a decision alternative (Complex Size) , then a state of nature follows (demand for the cond.) , and finally a consequence will occur . In this case , the consequence is the PDC's profit .

Comprehensive Example (Cont'd)

- Influence Diagram
 - ✓ An influence diagram is a graphical device that shows the relationship among the decision , the chance events , and the consequences for a decision problem .
 - ✓ The **nodes** in an influence diagram are used to represent the decision , chance events , and consequences .
 - ✓ Rectangles (or squares or boxes) are used to depict the **decision nodes** .
 - ✓ Circles or ovals are used to depict **chance nodes**.
 - ✓ Diamonds are used to depict the **consequence nodes** .
 - ✓ Lines connecting the nodes referred to as arcs , show the direction of influence that the node have on one another .

Influence Diagram



Payoff Tables

- Given the three decision alternatives and the two state of nature , which complex size should PDC choose ?
- To answer this question , PDC will need to know the consequence associated with each decision alternatives and each state of nature.
- In decision analysis , we refer to the consequence resulting from a specific combination of a decision alternatives and a state of nature as **Payoff**.

Payoff Table for the PDC cond. Project
(Payoffs in \$ Million)

State of Nature

Decision Alternatives	Strong Demand S1	Weak Demand S2
Small complex d1	8	7
Medium Demand d2	14	5
Large complex d3	20	-9

Decision Making Without
Probabilities

- ☺ **Optimistic Approach**
- ☹ **Conservative Approach (Maximum Regret)**
- ☹ **Minimax Regret Approach**

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Decision Making Without Probabilities

- Some approaches to decision-making do not require the decision-maker to formulate assumptions regarding probabilities for future states of nature. These can be used if the decision-maker just has no idea what the future probabilities might be, which is a perfect description of decision-making under conditions of **uncertainty**.
- ✓ These different approaches can lead to different decision recommendations. Therefore, if a decision-maker is going to use one of them, he needs to understand how each one works in order to select the most appropriate approach.

Optimistic Approach

- When evaluating decision alternatives using the **optimistic approach**, we evaluate them in terms of the **best** possible payoff that could occur. If maximum profit is the goal, a decision-maker using the optimistic approach would simply recommend whichever course would lead to the greatest possible profit.
- ✓ PDC has three Maximum values in the payoff table (8 , 14 , 20) and it will select d3 with 20 million , because it represent the Maximum of the maximum payoff values .

Conservative Approach

- As you can probably guess, the **conservative approach** is the opposite of the optimistic approach.
- The decision-maker will evaluate each alternative in terms of the **worst** possible payoff that could occur and recommend the one that provides the **best** of the worst possible payoffs. If maximum profit is the goal, the decision-maker will choose whichever course will lead to the greatest of the minimum profits possible for each alternative.
 - ✓ PDC has three minimum payoff (7,5,-9) and will select the best of the worst or the Maximum of the minimum payoff values , so it will select d1 with 7 million .

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Minimax Regret Approach

- Minimax Regret is an approach to decision making that is neither purely optimistic nor purely conservative . This will be illustrated in the following table called "Opportunity Loss or Regret) for every decision and the state of nature

Decision Alternatives	State of Nature	
	Strong Demand S1	Weak Demand S2
Small complex d1	12	0
Medium Demand d2	6	2
Large complex d3	0	16

Decision Alternatives	Maximum Regret
Small complex d1	12
Medium Demand d2	6 ←
Large complex d3	16

Select the minimum of the maximum regret

Decision Making With Probabilities

Decision Trees and Expected Value

Decision Trees and Expected Value

A **decision tree** is a means of determining the best course of action when there are several possible decision choices under a condition of **risk**.

Decision Trees and Expected Value

- A **decision tree** is a means of determining the best course of action when there are several possible decision choices under a condition of **risk**.
- Decision trees are used with probabilities to determine the expected value of the payoff of a project that may involve making several decisions.
- A decision tree depicts the natural or logical progression of events. Depending on the decision made at each decision point, the probabilities of the potential payoffs of that decision can be calculated in order to develop an overall expected value for the whole project.
- The decision tree is helpful for solving complex problems because it breaks them down into a series of smaller problems.

- In many decision-making situations, we can obtain probability assessments for the states of nature. When such probabilities are available, we can use the expected value approach to identify the best decision alternative. Let us first define the expected value of a decision alternative and then apply it to the PDC problem.
- Let
- N = the number of states of nature
- Because one and only one of the N states of nature can occur, the probabilities must satisfy two conditions:
- $P(S_j) \geq 0$ for all states of nature

$$\sum_{j=1}^N P(s_j) = P(s_1) + P(s_2) + \dots + P(s_N) = 1$$

- The **expected value** (EV) of decision alternative d_1 is defined as follows.

$$EV(d_i) = \sum_{j=1}^N P(s_j)V_{ij}$$

- In words, the expected value of a decision alternative is the sum of weighted payoffs for the decision alternative. The weight for a payoff is the probability of the associated state of nature and therefore the probability that the payoff will occur. Let us return to the PDC problem to see how the expected value approach can be applied .
- PDC is optimistic about the potential for the luxury high-rise condominium complex.
- suppose that this optimism leads to an initial subjective probability assessment of 0.8 that demand will be strong (s1) and a corresponding probability of 0.2 that demand will be weak (s2).

- Thus, $P(s_1) = 0.8$ and $P(s_2) = 0.2$. Using the payoff values in Table 4.1 and equation (4.4), we compute the expected value for each of the three decision alternatives as follows:
- $EV(d_1) = 0.8(8) + 0.2(7) = 7.8$
- $EV(d_2) = 0.8(14) + 0.2(5) = 12.2$
- $EV(d_3) = 0.8(20) + 0.2(-9) = 14.2$
- Thus, using the expected value approach, we find that the large condominium complex, with an expected value of \$14.2 million, is the recommended decision.

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- The calculations required to identify the decision alternative with the best expected value can be conveniently carried out on a decision tree. Figure 4.3 shows the decision tree for the PDC problem with state-of-nature branch probabilities. Working backward through the decision tree, we first compute the expected value at each chance node. That is, at each chance node, we weight each possible payoff by its probability of occurrence. By doing so, we obtain the expected values for nodes 2, 3, and 4, as shown in Figure 4.4.

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- Because the decision maker controls the branch leaving decision node 1 and because we are trying to maximize the expected profit, the best decision alternative at node 1 is $d3$.
- Thus, the decision tree analysis leads to a recommendation of $d3$ with an expected value of \$14.2 million. Note that this recommendation is also obtained with the expected value approach in conjunction with the payoff table.

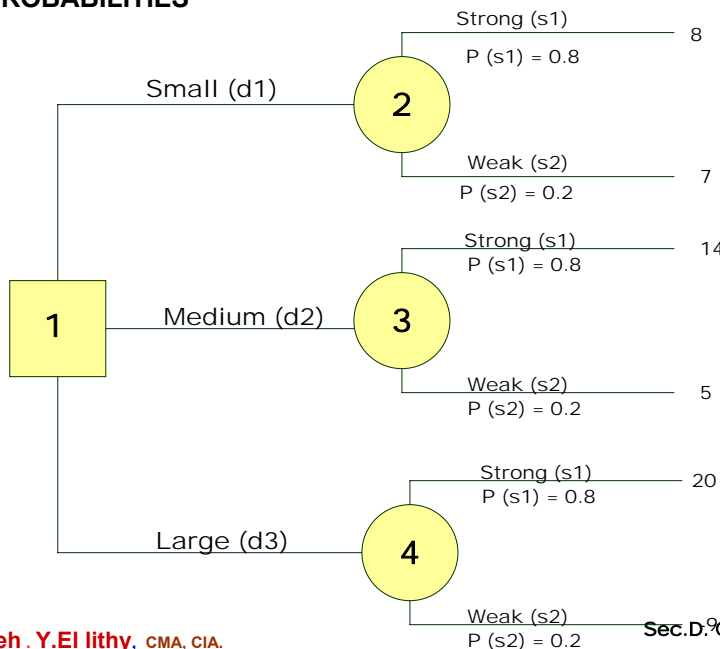
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- Other decision problems may be substantially more complex than the PDC problem, but if a reasonable number of decision alternatives and states of nature are present, you can use the decision tree approach outlined here. First, draw a decision tree consisting of decision nodes, chance nodes, and branches that describe the sequential nature of the problem.
- If you use the expected value approach, the next step is to determine the probabilities for each of the states of nature and compute the expected value at each chance node. Then select the decision branch leading to the chance node with the best expected value. The decision alternative associated with this branch is the recommended decision.

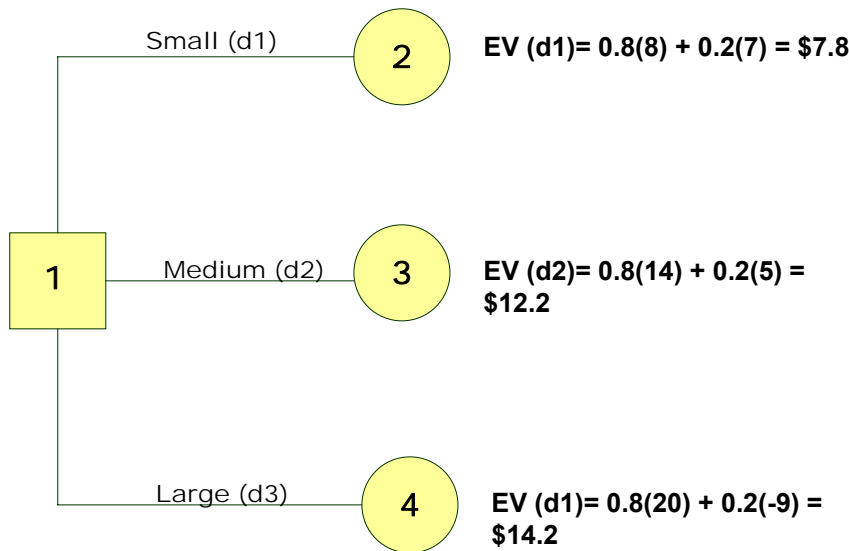
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➤ **Figure 4.3 PDC DECISION TREE WITH STATE-OF-NATURE BRANCH PROBABILITIES**



➔ Figure 4.4 APPLYING THE EXPECTED VALUE APPROACH USING DECISION TREES

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➔ **Expected Value of Perfect Information**

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- ➔ Suppose that PDC has the opportunity to conduct a market research study that would help evaluate buyer interest in the condominium project and provide information that management could use to improve the probability assessments for the states of nature. To determine the potential value of this information, we begin by supposing that the study could provide *perfect information* regarding the states of nature; that is , we assume for the moment that PDC could determine with certainty, prior to making a decision, which state of nature is going to occur.
- ➔ To make use of this perfect information, we will develop a decision strategy that PDC should follow once it knows which state of nature will occur. A decision strategy is simply a decision rule that specifies the decision alternative to be selected after new information becomes available.

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- To help determine the decision strategy for PDC, we have reproduced PDC's payoff table as Table 4.6. Note that, if PDC knew for sure that state of nature s_1 would occur, the best decision alternative would be d_3 , with a payoff of \$20 million. Similarly, if PDC knew for sure that state of nature s_2 would occur, the best decision alternative would be d_1 , with a payoff of \$7 million. Thus, we can state PDC's optimal decision strategy when the perfect information becomes available as follows:
 - If s_1 , select d_3 and receive a payoff of \$20 million.
 - If s_2 , select d_1 and receive a payoff of \$7 million.

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- What is the expected value for this decision strategy? To compute the expected value with we return to the original probabilities for the states of nature :
 - ✓ $P(s_1)=0.8$, and $P(s_2) = 0.2$.
- Thus, there is a 0.8 probability that the perfect information will indicate state of nature s_1 and the resulting decision alternative d_3 will provide a \$20 million profit. Similarly, with a 0.2 probability for state of nature s_2 , the optimal decision alternative d_1 will provide a \$7 million profit. Thus, from equation (4.4), the expected value of the decision strategy that uses perfect information is
 - ✓ $0.8(20) + 0.2(7) = 17.4$

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- We refer to the expected value of \$17.4 million as the *expected value with perfect information* (EVwPI).
- Earlier in this section we showed that the recommended decision using the expected value approach is decision alternative *d3*, with an expected value of \$14.2 million. Because this decision recommendation and expected value computation were made without the benefit of perfect information, \$14.2 million is referred to as the *expected value without perfect information* (EVwoPI).

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- The expected value with perfect information is \$17.4 million, and the expected value without perfect information is \$14.2; therefore, the expected value of the perfect information (EVPI) is $\$17.4 - \$14.2 = \$3.2$ million.
- In other words, \$3.2 million represents the , additional expected value that can be obtained if perfect information were available about the states of nature.

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➤ Generally speaking, a market research study will not provide "perfect" information; however, if the market research study is a good one, the information gathered might be worth a sizable portion of the \$3.2 million. Given the EVPI of \$3.2 million, PDC should seriously consider the market survey as a way to obtain more information about the states of nature.

F.4.6 : PAYOFF TABLE FOR THE PDC CONDOMINIUM PROJECT (\$ MILLION)

Decision Alternative	State of Nature	
	Strong Demand s 1	Weak Demand S2
Small complex, d_1	8	7
Medium complex, d_2	14	5
Large complex, d_3	20	-9

- In general, the expected value of perfect information is computed as follows:
- $EVPI = IEVwPI - EVwoPI$
- where
- EVPI = expected value of perfect information
- EVwPI = expected value *with* perfect information about the states of nature
- EVwoPI = expected value *without* perfect information about the states of nature

Other Quantitative Techniques

- Simulation
- Queuing theory
- Markov process
- Sensitivity analysis

SIMULATION

SIMULATION

- Simulation is one of the most widely quantitative approaches to decision making.
- It is a method for learning about a real system by experimenting with a model that represents the system.
- The simulation model contains the mathematical expressions and logical relationships that describe how to compute the value of the outputs given the value of the inputs .

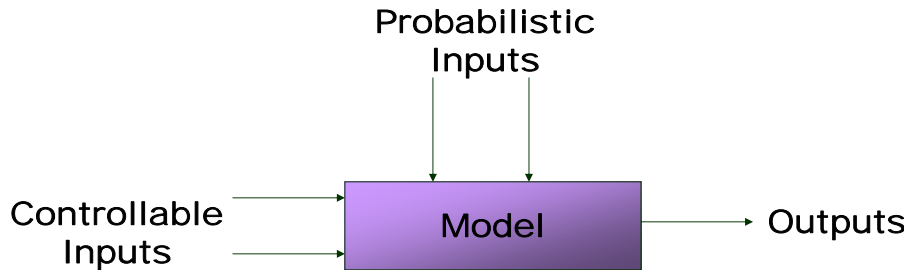
Inputs in Simulation

- Any simulation model has two inputs :
 - ✓ Controllable inputs
 - ✓ Probabilistic inputs
- In conducting a *simulation experiment* , an analyst selects the value , or values , for the **controllable inputs** . Then the values for the **probabilistic inputs** are randomly generated.

Outputs in Simulation

- The simulation model uses the values of the controllable inputs and the values of the probabilistic inputs to compute the value , or values , of the output .

Diagram of a Simulation Model



Learn from the Experiment

- By conducting a series of experiments using a variety of values for the controllable inputs , the analyst learns how values of the controllable inputs affect the or change the output of the simulation model .
- After reviewing the simulation results , the analyst is often able to make decision recommendations for the controllable inputs that will provide the desired output for the real system .

Example

➤ New Product Development

✓ The objective of this simulation is to determine the probability that a new product will be profitable . A model is developed relating profit (the output measure) to variety probabilistic inputs such as demand , parts cost , and labor cost .

➤ This example will be illustrated through Risk Analysis without simulation , then we will show a comprehensive Risk Analysis with Simulation .

RISK ANALYSIS WITHOUT SIMULATION

RISK ANALYSIS WITHOUT SIMULATION

- Risk analysis : is the process of predicting the outcome of a decision in the face of uncertainty , we will describe a problem that involves considerable uncertainty , the development of a new product .
- Portacom company introduce new product Printer HQ , preliminary marketing and financial analysis have provided the following selling price , first-year administrative cost , and first-year advertising cost .
 - ✓ Selling price = \$249 per unit
 - ✓ Administrative cost = \$400 000
 - ✓ Advertising cost = \$600 000

RISK ANALYSIS WITHOUT SIMULATION

(Cont'd)

- In the simulation model for the Portacom problem , the preceding values are constants and are referred to as parameters of the model .
- The cost of direct labor (DL) , the cost of parts (DM) , and the first-year demand (QD) for the printer are not known with certainty and are considered probabilistic inputs . At this stage of planning process , Portacom's best estimates of these inputs are
 - ✓ \$45 per unit for the DL
 - ✓ \$90 per unit for DM ,
 - ✓ Q 15000 unit for the first-year demand .

RISK ANALYSIS WITHOUT SIMULATION

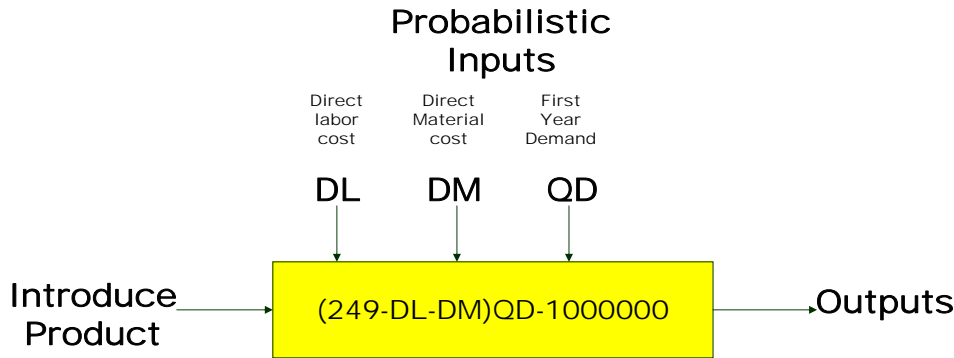
(Cont'd)

- Portacom would like an analysis of the first year-profit potential for the printer . Because of Portacom's tight cash flow situation , management is particularly concerned about the potential for a loss .
- One approach to risk analysis is called **WHAT-IF ANALYSIS** .

WHAT-IF ANALYSIS (Cont'd)

- Recall that PortaCom's estimates of the DL , DM , Q are \$45 , \$90 , and 15000 units respectively . These values institute the base-case scenario for PortaCom. Substituting these values into equation yields the following profit projection .
- Profit
 - ✓ = $(\$249 - 45 - 90)(15000) - 1000\ 000 = \$710\ 000$
- Thus the base case scenario leads to to an anticipated profit of \$710 000 .

PortaCom Profit Model



WHAT-IF ANALYSIS (Cont'd)

- In risk analysis we are concerned with both the probability of a loss and the magnitude of a loss . Although the base case scenario looks appealing , PortaCom might be interested in what happens if the estimates of the DL , DM , QD do not turn out to be less as expected under the base-case scenario
- For instance , suppose that PortaCom believes that DL cost could range from \$43 per unit to \$47 per unit , DM cost per unit could range from \$80 to \$100 per unit , and the first year QD could range from 1500 to 28500 units . Using these ranges what-if analysis can be used to evaluate worst-case scenario and a best-case scenario .

WHAT-IF ANALYSIS (Cont'd)

- **Worst-case scenario**
 - ✓ = $(\$249-47-100)(1500)-1000\ 000 = -\$847,000$
 - ✓ Leads to projected loss of $-\$847,000$
- **Best-case scenario**
 - ✓ = $(\$249-43-80)(28500)-1000\ 000 = \$2,591,000$
 - ✓ Leads to projected profit of $\$2,591,000$
- **At this point the what-if analysis provides the conclusion that profits can range from a loss of \$847,000 to a profit of \$2,591,000 with base-case scenario value of \$710,000 . Although the base-case profit of \$710,000 is possible , the what-if analysis indicates that either a substantial loss or a substantial profit is possible .**

WHAT-IF ANALYSIS (Cont'd)

- **Other scenarios that PortaCom might want to consider can also be evaluated . However , the difficulty with what-if analysis is that it does indicate the likelihood of the various profit or loss values . In particular , we do not know anything about the probability of a loss.**
- **Simulation assess the probability of a profit and the probability of a loss .**

SIMULATION

SIMULATION

- Using simulation to perform risk analysis for the PortaCom problem is like playing out many what-if scenarios by randomly generating values for the probabilistic inputs.
- The advantage of simulation is that it allows us to assess the probability of a profit and the probability of a loss .
- Using what-if approach to risk analysis , we selected values for the probabilistic inputs (DL cost per unit , DM cost per unit , QD first year demand) , and then computed the resulting profit .

SIMULATION (Cont'd)

- Applying simulation to the PortaCom problem requires generating values for the probabilistic inputs that are representative of what we might observe in practice . To generate such values , we must know the probability distribution for each probabilistic input .

SIMULATION (Cont'd)

- Further analysis by PortaCom has led to the following distributions for DL cost per unit , DM cost per unit , and the first-year demand QD :
 - ✓ DL Cost with **discrete** Probability Distribution
 - Tested in the CMA exam , asking for the intervals.
 - ✓ DM Cost with Uniform PD
 - Out of the scope of the CMA exam.
 - ✓ Demand with Normal PD
 - Out of the scope of the CMA exam.

DL Cost with discrete PD

- DL cost : PortaCom believes that DL cost will range from \$43 to \$47 per unit and is described by the discrete probability distribution shown below table no.1 :

DL cost per unit	Probability
\$43	0.1
\$44	0.2
\$45	0.4
\$46	0.2
\$47	0.1

The highest probability of 0.4 is associated with a DL cost of \$45 per unit .

DM Cost with Uniform PD

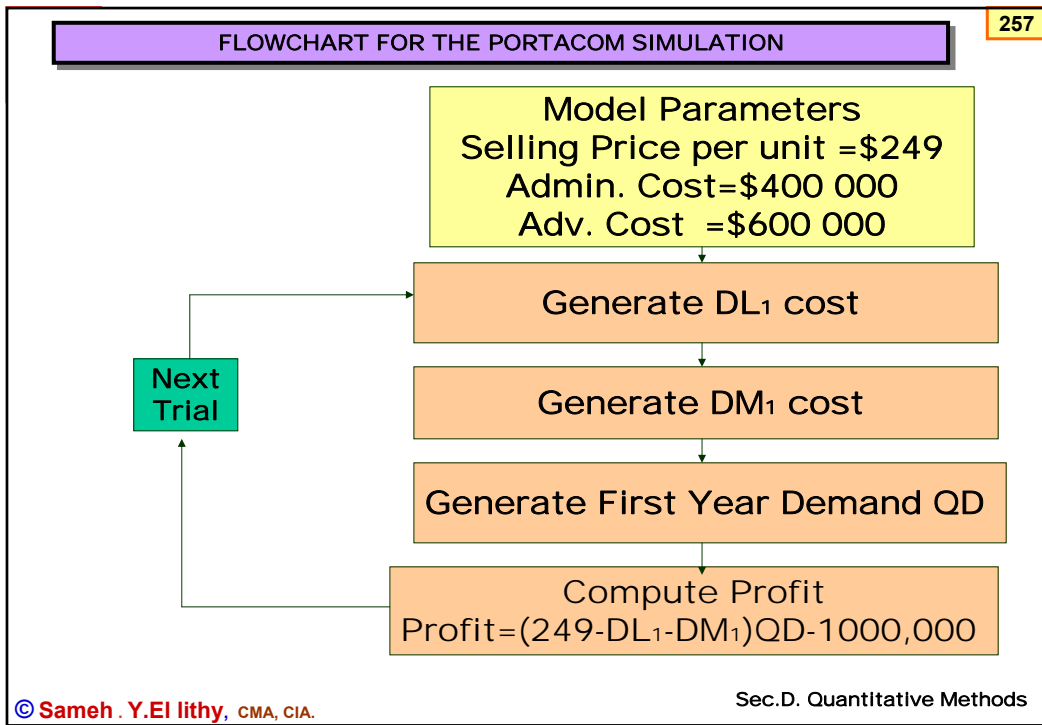
- This cost depends upon the general economy , the overall demand for parts , and the pricing policy of PortaCom's parts suppliers. PortaCom believes that the parts cost will range from \$80 to \$100 per unit and is distributed by the uniform probability distribution , in other words DM costs per unit between \$80 to \$100 are equally likely .

Demand with Normal PD

- PortaCom believes that first-year demand is described by the normal probability distribution .
- The expected value (mean) of the first year demand is 15,000 units , with standard deviation 4,500 units .

Generation of the values of Probabilistic Inputs

- To simulate the PortaCom problem , we must generate values for the three probabilistic inputs , and compute the resulting profit . Then , we generate another set of the probabilistic inputs , compute a second value for profit , and so on . We continue this process until we are satisfied that enough trials have been conducted to describe the probability distribution for profit . This process of generating probabilistic inputs and the value of output is called simulation .



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RANDOM NUMBERS AND GENERATING PROBABILISTIC INPUT VALUES

- An essential part of the simulation procedure is the ability to generate representative values for the probabilistic inputs .
- In the PortaCom simulation , representative values must be generated for the direct labor cost per unit DL , the parts cost DM cost per unit , and the first year demand QD .
- Random numbers and the probability distributions associated with each probabilistic input are used to generate representative values.

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RANDOM NUMBERS AND GENERATING
PROBABILISTIC INPUT VALUES (Cont'd)

- To illustrate how to generate these values , we need to introduce the concept of *computer-generated random numbers*.
- Computer-generated random numbers are randomly selected from simulation software or Excel application , these decimal numbers from 0 up to , but not including , 1 . The computer-generated random numbers are equally likely and are uniformly distributed over the interval from 0 to 1 .

RANDOM NUMBERS AND GENERATING
PROBABILISTIC INPUT VALUES (Cont'd)

- We begin by showing how to generate a value for the DL cost per unit . The approach described is applicable for generating values from any discrete probability distribution .
- An interval of random numbers is assigned to each possible value of the DL cost in such a fashion that the probability of generating a random number in the interval is equal to the probability of the corresponding DL cost . The next table shows how this process is done .

**RANDOM NUMBERS AND GENERATING
PROBABILISTIC INPUT VALUES (Cont'd)**

- The interval of random numbers 0.0 but less than 0.1 is associated with a DL cost per unit of \$43 , the interval of random numbers 0.1 but less but 0.3 is associated with a DL of \$44 and so on .
- Each trial the generated number will determine the DL cost per unit that will set for that trial . If the random number is 0.0 but less than 0.1 , we set the DL cost equal to \$43 . If the random number is 0.1 but less 0.3 , we set the DL cost equal to \$44 , and so on .

**RANDOM NUMBERS AND GENERATING
PROBABILISTIC INPUT VALUES (Cont'd)**

- Each trial of the simulation requires a value for the DL cost . Suppose that in the first trial the random number is 0.9109 . From the previous table , the simulated value for the DL cost is \$47 per unit . Suppose that on the second trial the random is 0.2841 . From , the simulated value for the for the DL cost is \$44 per unit .
- The next table shows the results obtained for the first 10 simulation trials .

**Random Number Intervals For Generating Values Of
DL cost Per Unit (table no.2)**

DL cost per unit	Probability	Interval of Random Numbers
\$43	0.1	0.0 but less than 0.1
\$44	0.2	0.1 but less than 0.3
\$45	0.4	0.3 but less than 0.7
\$46	0.2	0.7 but less than 0.9
\$47	0.1	0.9 but less than 1.0

**Random Generation of Ten Values For The DL
cost Per Unit (table no.3)**

Trial	Random Numbers	DL Cost (\$)
1	0.9109	47
2	0.2841	44
3	0.6531	45
4	0.0367	43
5	0.3451	45
6	0.2757	44
7	0.6859	45
8	0.6246	45
9	0.4936	45
10	0.8077	46

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PortaCom Simulation Results For Ten Trials (table no.4)				
Trial	DL cost Per Unit (\$)	DM cost Per Unit (\$)	Unit Sold	Profit(\$)
1	47	85.36	17,366	1,025,570
2	44	91.68	12,900	461,828
3	45	93.35	20,686	1,288,906
4	43	98.56	10,888	169,807
5	45	88.36	14,259	648,911
6	44	94.68	22,904	1,526,769
7	45	88.65	15,732	814,686
8	45	82.37	17,804	1,165,501
9	45	93.89	5,902	-350,131
10	46	95.74	12,918	385,585
Total	449	912.64	151,359	7,137,432
Average	44.90	91.26	15,135	\$713,743

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Running the Simulation Model

- The simulated profit for the PortaCom printer if the DL cost is \$47 per unit , the DM cost is \$85.36 per unit , and first-year demand is 17,366 units is \$1,025,570 . Of course , one simulation trial does not provide a complete understanding of the possible profit and loss . Because other values are possible for the probabilistic inputs , we can benefit from additional simulation trials .
- Suppose that on a second simulation trial , random numbers of 0.2841 , 0.5842 , and 0.3204 are generated for the DL cost , DM cost , and first –year demand , respectively . These random numbers will provide the probabilistic inputs of \$44 for the DL cost , \$91.68 for DM cost , and 12,900 for first –year demand . These values provide a simulated profit of \$461,828 on the second simulation trial .

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Running the Simulation Model (Cont'd)

- Repetition of the simulation process with different values for the probabilistic inputs is an essential part of any simulation . Through the repeated trials , management will begin to understand what might happen when the product is introduced into the real world . We have shown the results of ten simulation trials in table no. 4. For these 10 cases , we find a profit as high as \$1,526,769 for the 6th trial and a loss of \$350,131 for the 9th trial.
- Thus we see both the possibility of a profit and a loss .
- Average for the ten trials are presented at the bottom of the table . We see that the average profit for the ten trials is \$713,743 . The probability of a loss is 0.10 , because one of the ten trials (the 9th) resulted in a loss . We note also that the average values for labor cost , DM cost , and first-year demand are fairly close to their means of \$45 , \$90 , 15000 , respectively .

Output from Simulation

- In practice the summary statistics that generated from simulation include the following items :
 - ✓ Mean Profit
 - ✓ Standard Deviation
 - ✓ Minimum Profit
 - ✓ Maximum Profit
 - ✓ Number of losses
 - ✓ Probability of loss

Monte Carlo Simulation

- ➔ The PortaCom simulation model is based on independent trials in which the results for one trial do not affect what happens in subsequent trials . Historically , simulation studies such as this were referred to as *Monte Carlo Simulation* . The term *Monte Carlo Simulation* was used because early practitioners of simulation saw similarities the models they were developing and the gambling games played in the casinos of Monte Carlo. Today many individuals the term Monte Carlo simulation more broadly to mean any simulation that involve randomly generating values for the probabilistic inputs .

Typical applications of Simulation

- ➔ AIR LINE OVERBOOKING
- ➔ INVENTORY POLICY
- ➔ TRAFFIC FLOW
- ➔ WAITING LINES

Benefits and Limitations of Simulation

- ➔ Simulation is very flexible and can be used for a wide variety of problems. Simulation is easily understood, and thus management more readily accepts its results. Many simulation models can be implemented without special software packages, because most spreadsheet packages provide useable add-ins. For more complex problems, simulation applications are available.
- ➔ However, simulation is not an optimization technique. It is a method that can predict how a system will operate when certain decisions are made for controllable inputs as well as when randomly generated values are used for the probabilistic inputs. Simulation can be effective for designing a system that will provide good performance; but there is no guarantee that it will be the best possible performance.

Benefits and Limitations of Simulation (Cont'd)

- ➔ Furthermore, the results will be only as accurate as the model that is used. A poorly developed model or a model that does not reflect reality will provide poor results and may even be misleading. And there is no way to test whether the assumptions and relationships used are correct without the passage of time.

Waiting Lines or Queuing Theory

On the CMA Exams. Knowledge of when it is applicable is sufficient.
Just an **overview level**

Waiting Lines or Queuing Theory

- ➔ To provide good service, a company needs to minimize the amount of time its products (or its customers) are waiting in lines. However, this desire to minimize the cost of waiting must be counterbalanced with the cost of creating more capacity to move the items through the line faster.
- ➔ **Queuing theory** is the process of determining the most efficient and effective way to move people or goods through a line, keeping waiting times to a minimum in the most economical manner. Waiting line models consisting of mathematical formulas can be used to make decisions that balance desired levels of service with the costs of providing the service.

Typical Application of Queuing (waiting-line) theory

- Examples of queuing systems
 - ✓ Bank teller windows
 - ✓ Grocery checkout counters
 - ✓ Airport holding patterns
- Examples of the application of queuing theory would be in restaurants, stores, airports (both passengers and airplanes coming and going), traffic lights and the like.

Markov Analysis

On the CMA Exams. Knowledge of when it is applicable is sufficient.
Just an **overview level**.

Markov Analysis

- Markov process models in general are used to study systems and changes that take place in the systems during repeated trials.
- **Markov analysis** does not attempt to optimize any part of the system. Instead, it is intended to describe the future behavior of the system. This information is used to make decisions, because it can help determine the probabilities for the occurrence of certain events.

Markov Analysis (Cont'd)

- One type of Markov analysis is called **Markov chains with stationary transition probabilities**. The “stationary transition probability” is the probability of something happening again tomorrow if the same thing happened today.
 - ✓ This type of analysis is used to determine the probability that a machine that is functioning in the current period will continue to function in the next period or whether it will break down. Or it could be used to determine the probability that a customer of a store during a particular month will also be a customer of that store during the following month.

Markov Analysis (Cont'd)

- Markov analysis of this type uses only input from the most recent period to predict events in the next period. For that reason it is said to have a “**memoryless property**,” because the current state of the system plus the transition probabilities are the only information needed to predict future occurrences. What happened before the current period is irrelevant.

Typical Application of Markov Analysis

- Markov analysis works well for
 - ✓ Making a forecast of market shares for competitors,
 - ✓ Analyzing machine breakdowns,
 - ✓ Planning the movement of patients in hospitals,
 - ✓ Determining subscription duration and
 - ✓ Describe the probability that a consumer purchasing brand A in one period will purchase brand B in the next period .
 - ✓ Marketing Application that involve an analysis of the store – switching behavior of supermarket customers.

Sensitivity (Postoptimality)Analysis

Sensitivity (Postoptimality)Analysis

- After a problem has been formulated into any mathematical model, it may be subjected to sensitivity analysis. This approach is especially useful and significant when probabilities of states of nature and decision payoffs are derived subjectively rather than by using objectively quantifiable information.

Typical Application of SA

- In linear programming problems, sensitivity is the range within which a constraint value, such as a cost coefficient or any other variable, may be changed without changing the optimal solution. Shadow price is the synonym for sensitivity in that context.
- In the application of discounted cash flow methods (e.g., net present value), a sensitivity analysis might be performed to ascertain the effects of variability of the discount rate or periodic cash flows.

Typical Application of SA (Cont'd)

- Financial planning models, including those for cash flows and capital budgeting, are other significant applications of sensitivity analysis. For example, changes in selling prices or resource costs may affect available cash and require more or less short-term borrowing.
- The calculation of the margin of safety in a CVP analysis.

