Investments

Lecture Notes

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Syllabus: Course Description

- In this course you will study the theory and practice of investment management in domestic and global financial markets.

- The course comprehensively describes conceptual paradigms and their extensive applications in practice.

Syllabus: Contents

- The following topics will be covered:
  - Asset allocation;
  - Security selection;
  - ETFs: an easy way to invest;
  - Computing rate of return on various investments;
  - Short selling and buying on margin;
  - The risk-return tradeoff;
  - The stock versus bond performance;
  - Constructing price weighted indexes (e.g., Dow Jones 30, Nikkei 225) as well as value weighted indexes (e.g., TA25, S&P500, NASDAQ);
  - Recovering key financial ratios from financial statements;
  - Debating market efficiency;

Syllabus: Contents

- Analyzing performance of mutual funds;
- Comparing mutual funds to hedge funds;
- Examining evidence on market anomalies, especially, the size, value, price momentum, earnings momentum, and volatility effects in stock prices;
- Diversification gain and portfolio volatility;
- Forming optimal portfolios with constraints;
- Investing in public and private equities, commodities, and bonds;
- Understanding and pricing derivatives securities;
- Valuing equities using the discounted cash flows approach and price multipliers.

Syllabus: Prerequisite

- Make sure you have taken all the prerequisite courses.
- The course requires adequate analytical skills.
- Familiarity with statistics should extend through concepts of mean, standard deviation, covariance, correlation, the normal distribution, and regression analysis.
- A good grounding in Excel is essential for solving the case studies as well as analyzing traditionally applied paradigms in financial economics.

Syllabus: Resources

- *Textbook*: The class notes are fairly comprehensive. If you wish to enhance your knowledge, you can use the following textbooks:
  - *Fundamentals of Investments Valuation and Management* by Jordan & Miller.
  - *Investments* Bodie, Kane, and Marcus
- *TA:* Lior Metzker; email: lior.metzker at mail.huji.ac.il

Syllabus: Grading

- **Assignments (36%)**: There are three case studies as well as frequent problem sets corresponding to end-of-chapter questions. You can form groups of up to (**no more than**) four students to prepare the cases. However, end-of-chapter questions must be solved individually. Each case accounts for 9% of the final grade. All problem sets combined also account for 9% of the final grade.

- **Final Exam (64%)**: The final exam will be based on the material covered in class (in letter and spirit), class handouts, class discussions, examples, case studies, and assigned readings. The exam is closed books and closed notes. However, you can bring in one piece of paper with handwritten or printed notes (double-sided, A4 size). You are not allowed to use any other notes. You can also use a calculator during the exam.

Syllabus: Class Attendance

- To get credit for this course it is **mandatory** to attend all the sessions. You are responsible for any announcement (including due dates for the case studies and problem sets), discussion, and remarks made in class.

- **A nontrivial fraction of the final exam questions could be based on class discussion, coverage of recent events in financial markets, and examples which are uncovered in the lecture notes.**

Case 1: Dimensional Fund Advisers (DFA)

http://hbr.org/product/dimensional-fund-advisers-2002/an/203026-PDF-ENG

- In 2002, the time when the case study was written, DFA was a $30 billion fund. Ever since there has been a phenomenal growth in assets managed by DFA. See updated figures in the DFA website.
- You will find in the case study that several features make DFA an unusual family of mutual funds. In particular, DFA believes in market efficiency. It hence relies on passive strategies undertaking more of a buy-and-hold investment approach than active search for mispriced stocks. DFA uses academic research to form investments and assess their performance. It has also specialized in trading large blocks of small stocks at discount prices.

Case 1: Dimensional Fund Advisers (DFA)

- The case covers several topical subjects in finance including efficient markets, models of capital market equilibrium (e.g., the CAPM), the Fama-French three-factor model, financial instruments, investment management, tax management, liquidity, and stock trading.

DFA - Questions

- **Q1**. Are DFA investment management practices consistent with market efficiency?
- **Q2**. What are the Fama-French (FF) findings? How do Fama and French explain the size and book to market effects in stock prices? Looking forward, should you expect small stocks to outperform large stocks? Value stocks to outperform growth stocks? Do FF findings hold in other financial markets beyond the US? What are the alternative (non-risk) explanations for the size and value effects?
- **Q3**. Visit http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html Download returns on 25 size and book to market portfolios and then replicate the table in the class notes (the one in the value anomaly section) based on:
  a. The entire sample period starting from 1/70
  b. The 1/1/90-31/12/1999 period
  c. The 1/1/2000-31/12/2010 period
  d. The period starting from 1/1/2011.

Describe the size and book-to-market effects in all these periods.

DFA - Questions

- **Q4**. What are liquidity constraints on trading? How does DFA deal with liquidity constraints, and what are the costs and benefits of implementing the DFA's strategy? Has the DFA's liquidity proving strategy been improving its performance over the years?
- **Q5**. Who are the most prominent competitors of DFA in the mutual fund industry? How could you explain the impressive progress DFA has made over time in attracting new money relative to its peers?
- **Q6**. Compare DFA performance to its major competitors. That is, pick at least two mutual funds managed by DFA (based on style or sector) and examine whether those funds have outperformed similar funds managed by at least two of DFA's competitors. Also examine whether the DFA funds have outperformed their benchmarks. You can use yahoo finance at http://finance.yahoo.com/ to get real time data.

DFA – Guidelines

- **Q1**: DFA has been closely affiliated with Eugene Fama - a Nobel price laureate from the University of Chicago. Fama has been a strong believer in market efficiency. It should be noted that also Robert Shiller won the Nobel Prize in 2013 along with Fama. Shiller believes that markets are inefficient and subject to behavioral biases. Who is right? This seems to be a philosophical question. The empirical evidence is not conclusive.

  Per Fama, if high book-to-market small-cap stocks earn more over the long run – they must be more risky and moreover the CAPM does not capture these additional risk sources. DFA claims to have adopted Fama's beliefs. Based on the case study, is it really clear that the DFA management believes in market efficiency?

DFA – Guidelines

- **Q2**: This question is about describing the Fama-French findings based on past evidence and future prospects. Here is some background. The Fama-French three-factor model (which extends the single-factor CAPM) has gained prominence in the industry and academia. Some folks do not accept that model. What could be the objections here? For one, the model followed the data and is not inspired by economic theory. There are also behavioral (non risk) explanations for the value premium. Here is one. Investors classify stocks to growth or value categories based on historical growth rates. Such investors are willing to pay high prices for growth stocks believing that the previously observed phenomenal growth rates will last for the very long run. But in reality, high growth rates are typically not long lasting – past high growth rate firms could become low growth rate firms in the future, and vice versa. Therefore, growth stocks earn less in the future once investors realize that growth firms perform poorly relative to prior expectations. That means the value premium could reflect investors' incorrect extrapolation of past growth rates. There are other behavioral explanations.

DFA – Guidelines

- **Q3**: This one is self-explanatory. Simply download the data and compute statistics similar to those displayed in the class notes. You are also requested to analyze patterns in the size and value effects during the distinct sample periods.
- **Q4**: DFA has established an excellent reputation in the niche of liquidity providing. What is the liquidity providing strategy, how does it work and what are the pros and cons? If you have anything else to say about the important concept of liquidity please feel free to detail it in your submitted case.
- **Q5**: Compare the growth of DFA to distinguished competitors (e.g., Vanguard, Fidelity, among other competitors) – has DFA attracted more inflows? If yes, how could you explain that.
- **Q6**: Identify competing mutual funds and compare performance.

Case 2: AQR's Momentum Funds (2012)

Please note that there are 2 links.

http://hbr.org/product/aqr-s-momentum-funds-a/an/211025-PDF-ENG

http://hbr.org/product/aqr-s-momentum-funds-b/an/211075-PDF-ENG?Ntt=AQR%27s%2520Momentum%2520Funds

This case study discusses the launch of several new retail mutual funds that offer investors the exposure to price momentum, a prominent investment style. While momentum strategies have been commonplace among hedge funds, the new AQR funds would become the first retail funds to focus on this strategy.

AQR's Questions

- **Q1**: Describe in general the momentum trading strategy. What are the common explanations for the existence of momentum in asset prices?
- **Q2**: Identify one of AQR momentum funds. Compare performance of that fund relative to the market portfolio?
- **Q3**: Describe potential reasons for the momentum crash during 2009.

AQR's Questions

- **Q4**: Can a retail investor easily imitate a momentum fund? Explain in detail whether a similar home-made strategy could be established.

- **Q5**: Compare two momentum funds – one for small cap stocks while the other for large cap stocks. Which one you would favor and why?

- **Q6:** There are also sector funds specializing in momentum. PTF implements momentum among technology stocks; PXI implements momentum among energy stocks. Explain the basic strategies of such funds (ETFs). Find other sector ETFs investing in momentum. What would be the benchmarks for assessing performance of such ETFs? Have momentum ETFs been able to beat their benchmarks?

- **Q7**: What can you learn about market efficiency based on the performance of momentum funds?

Case 3: Harvard Management Company (HMC)

http://hbr.org/product/harvard-management-company-2010/an/211004-PDF-ENG?Ntt=211004

The central issue in this case is to propose an asset allocation policy for Harvard Management Company using concepts of mean variance optimization and portfolio constraints studied in class. Let us start with the policy portfolio displayed in Exhibit 4. The policy portfolio is the long run asset mix of Harvard. It specifies the “neutral weighting” for each asset class. HMC was given a minimum and maximum range for each asset class within which they could trade. HMC made tactical asset allocation bets from time to time attempting to beat the policy portfolio in anticipation for short-term market moves. Some of the questions below involve using the Excel Solver for forming optimal portfolios under constraints for each of the asset classes under consideration.

HMC – Questions

- **Q1**: Given figures in Exhibits 4 and 17 what is the expected return and volatility of the policy portfolio?

- **Q2**: Find an efficient portfolio having the same expected return as the policy portfolio but lower volatility based on portfolio constraints displayed in Exhibit 18. Report the investment weights of this portfolio as well as its volatility.

- **Q3**: Find an efficient portfolio having the same volatility as the policy portfolio but higher expected return based on the same portfolio constraints. Report the investment weights of this portfolio as well as its expected return.

- **Q4**: Repeat questions 2 and 3 using the new set of constraints in Exhibit 19.

HMC – Questions

- **Q5**: Plot on one space two efficient frontiers – the first one pertains to the constraints in Exhibit 18 along with the inputs in Exhibit 4; the second one pertains to the constraints displayed in Exhibit 19 along with the inputs in Exhibit 4. Start the frontier from the Global Minimum Variance Portfolio (GMVP); end the frontier with the maximum expected return portfolio. Pick three other intermediate portfolios. Compare investment opportunities under the two scenarios – which one dominates and why? Why would one impose portfolio constraints to begin with?

- **Q6**: Look at the Historical Asset Mix in Exhibit 3 – how would you explain the vast change in the asset mix for the periods 1992 through 2010 (e.g., equities versus bonds versus commodities)? In your opinion, does the asset mix simply follow past changing market conditions or does it predict future market conditions?

HMC – Questions

- **Q7**: Look at the percentage of assets invested with internal and external managers in Exhibit 5 – how would you explain the change in that percentage over time?

- **Q8**: What are the fallacies of the mean variance paradigm?

- **Q9**: Some practitioners believe that the mean variance setup should be replaced with alternative mechanisms. For instance, consider an investor who maximizes expected return subject to some level of shortfall probability (probability to realize an investment return below the risk free rate). Shortfall probability is an example of a down size risk measure. What is down side risk? Bring other examples of down side risk measures. How is a down side risk measure different from stock return volatility? Why would a down side risk based investment design make sense for a loss averse investor?

HMC – Hints

Here are some guidelines for solving some of the questions. There is no need to attach excel files. The first question is merely a computational one. The objective here is to examine your ability to compute expected return and volatility of a portfolio accommodating multiple asset classes. Moreover, inputs from the first question will be used to solve the three questions that follow. In particular, the second (third) question requires displaying the weights of an efficient portfolio dominating the policy portfolio along the volatility (expected return) dimension. Of course, you are also required to exhibit the mean and volatility of these two dominating portfolios. So far everything is just computations. But questions (2) and (3) suggest that the policy portfolio is inefficient. Hence, why does HMC spend tremendous efforts on establishing the policy portfolio? Do your best efforts to answer this important question. Question (4) and (5) call for some new computations. You will see that with a tighter set of portfolio constraints the investment opportunities appear less attractive. Then explain: why imposing constraints to begin with? The fifth question is an open-ended one.

Investing in Securities Markets

Security Types

Basic Types	Major Subtypes
Equity	Stocks
Interest Bearing	Money market instruments; Bonds; Mortgages
Derivatives	Options, Futures, Swaption, FRAs, CDS, Eurodollars

Asset Allocation

- How should an investor allocate funds across the major asset classes: equities, fixed-income instruments, commodities, currencies, and cash and cash equivalents?

- Asset allocation aims to balance risk and reward based on our risk tolerance, taxation, liquidity, and investment horizon.

- The consensus among finance professionals is that asset allocation is the most crucial decision made by investors.

The State of Asset Allocation

- Indeed, various studies have shown that asset allocation accounts for more than 90% of the investment rate of return, while less than 10% originates from particular choices of stocks or bonds.

- But the profession lacks good enough models to communicate reliable guidance about how to optimally invest our wealth in major asset classes.

- While the mean-variance paradigm attempts to deliver a tractable modeling approach, it has nontrivial caveats, to be discussed later.

Security Selection

- Once you decide upon a policy portfolio which invests in stocks, bonds, commodities, currencies, and cash (e.g., 30%, 20%, 30%, 15%, and 5%) you have to pick the particular securities within each category.

- Stocks: big versus small market cap; growth versus value; high versus low past return; quality versus junk; high versus low credit risk; industry affiliations such as biotech, financial, oil, real estate, automobile, airlines, pharma, etc.

- Bonds: Government versus corporate; high versus low credit risk; long versus short duration; indexed versus nominal; fixed versus floating rates.

Examples of Security Selection

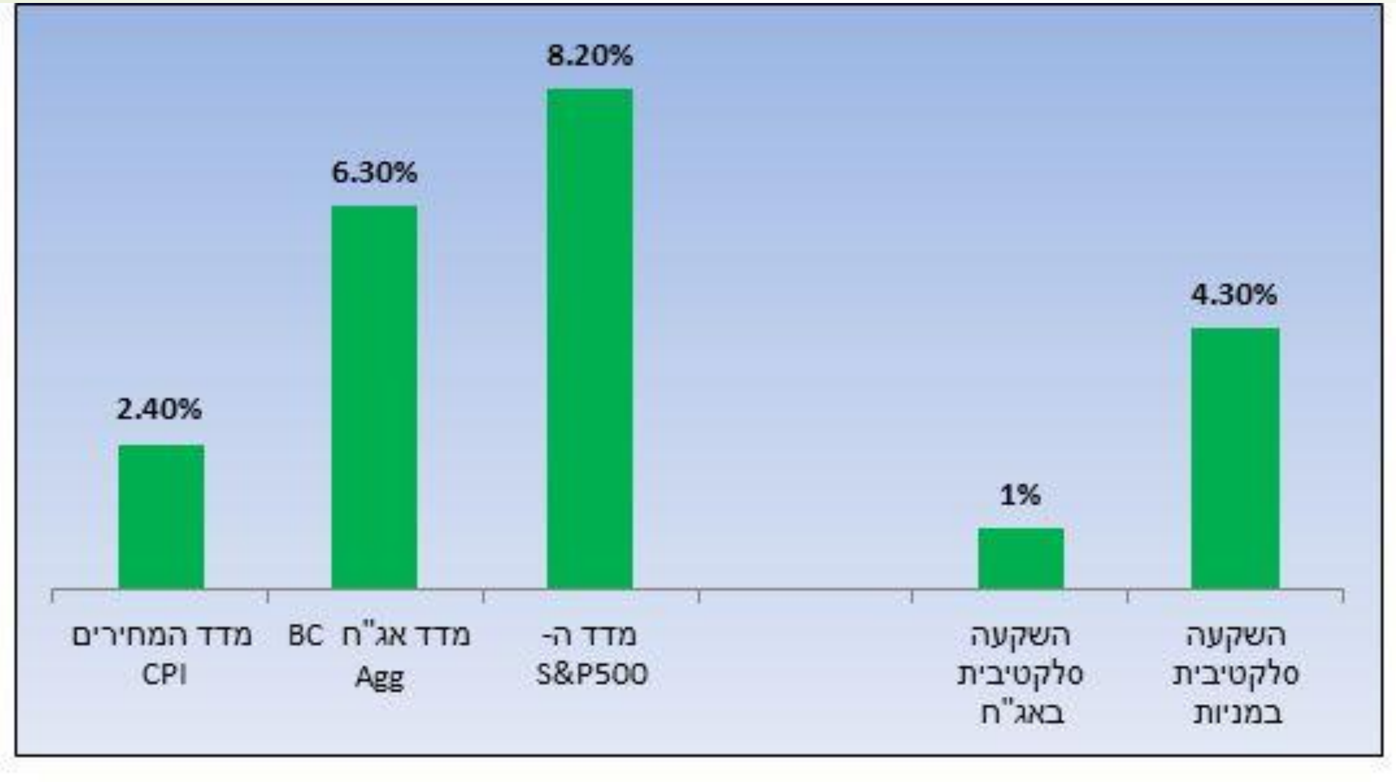
- Commodities: Gold, Oil, Soybeans, Citrus, etc.
- Currencies: Euro, Dollar, Yen, Pound, etc.
- Cash: money market funds versus bank CDs, etc.
- Advice: do not pick stocks based on emotions, fundamental analysts' recommendations, rumors, etc. Most likely you will underperform!!!

Asset Allocation VS. Stocks Picking

- Dalbar research company compared Senior Investment Managers versus benchmarks.
- The comparison was made between the years 1993 and 2012.
- It was found that even senior well-positioned managers had realized considerably lower returns than benchmarks.
- The managers dedicated great efforts to identify stocks and bonds that, in their view, were cheap.
- But they did not pay enough attention to asset allocation.

Asset Allocation VS. Stocks Picking

- The following graph makes the case.
- Benchmarks are on the left; active investments on the right.



The Process of Investing

- So avoid stock-picking, which typically leads to over trading and bad performance.
- By trading, you compete against some of the brightest minds in the world, while fending off the piranhas of trading costs.
- You can invest in the market index, using ETFs.
- You can invest in market anomalies (to be discussed later), using ETFs.
- You can follow some particular sectors of interest, again using ETFs.
- What is an ETF?

ETF

- An **exchange-traded fund (ETF)** is an investment fund traded on stock exchanges, much like stocks.

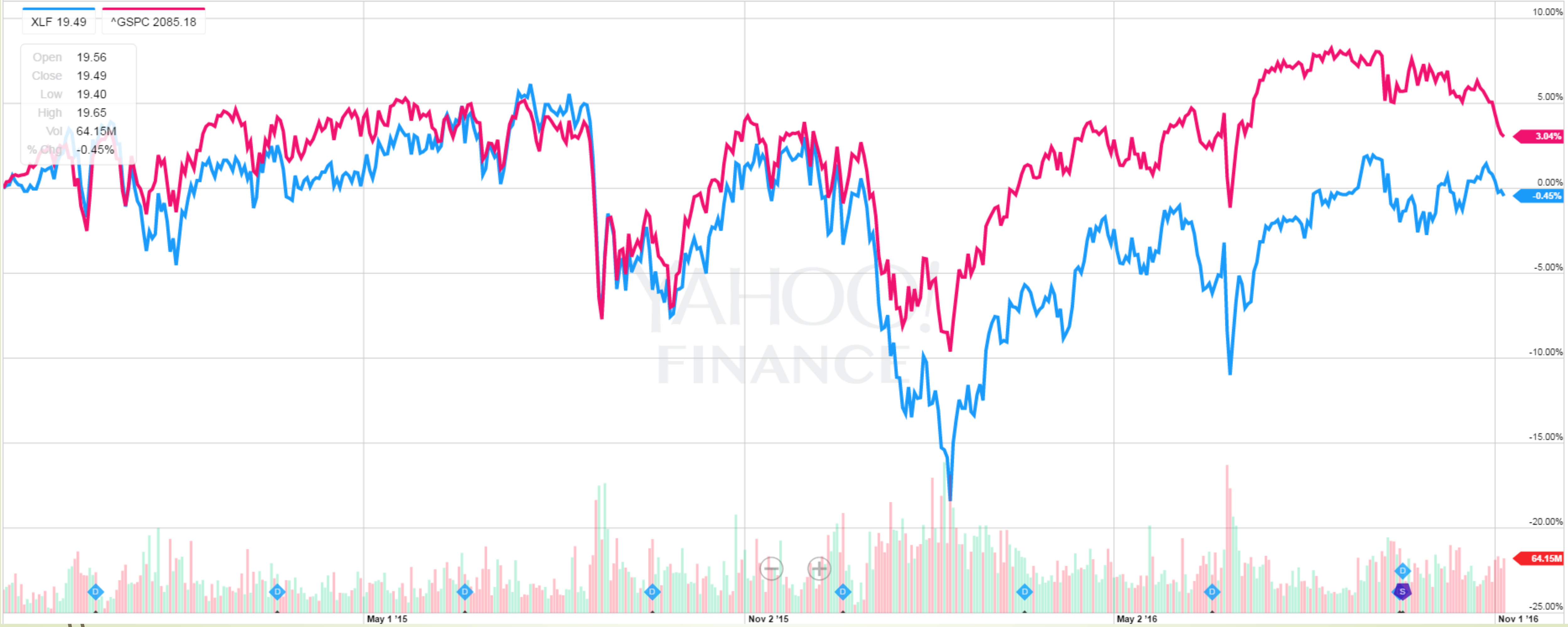
- An ETF holds assets such as stocks, commodities, or bonds, and trades close to its net asset value (NAV) over the course of the trading day.

- ETFs may be attractive due their low costs and fairly high liquidity.

- Today the net amount of assets managed by ETFs exceeds $3 trillion and it is expected to grow much higher.

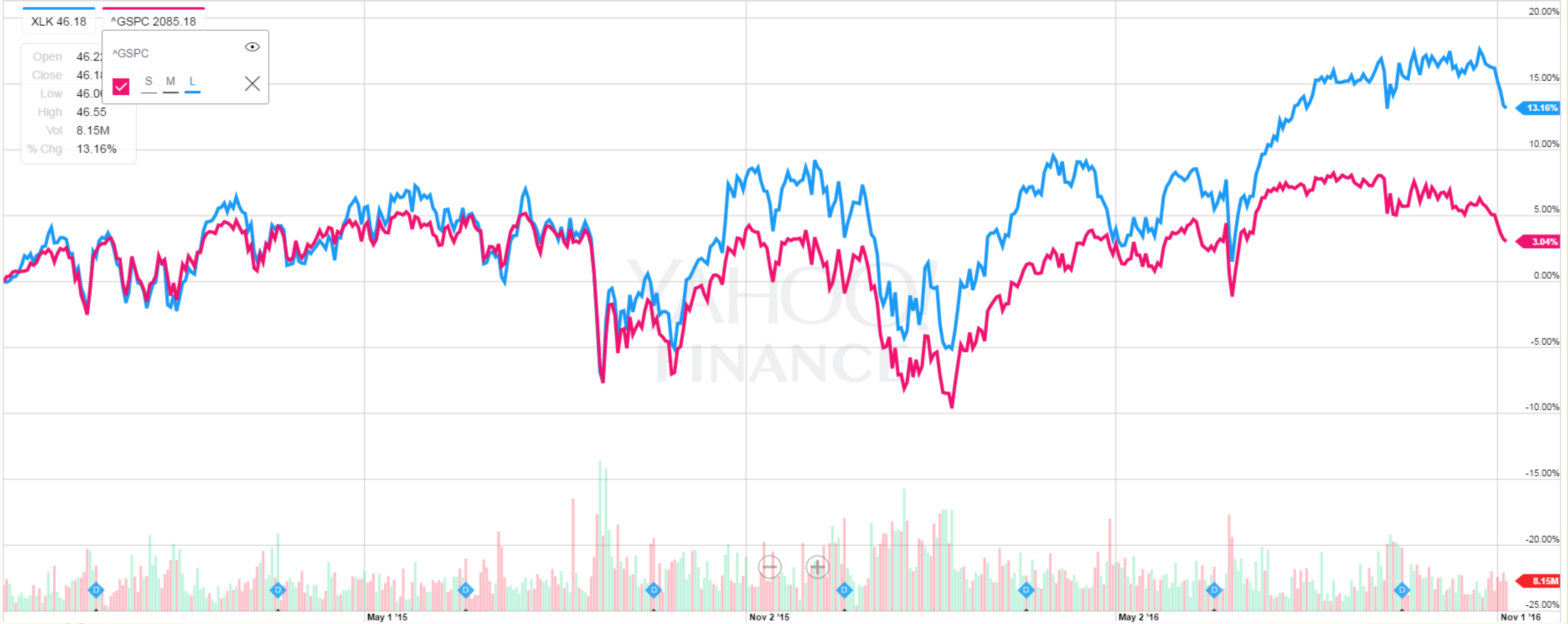
USA: Investing in Financials

- XLF



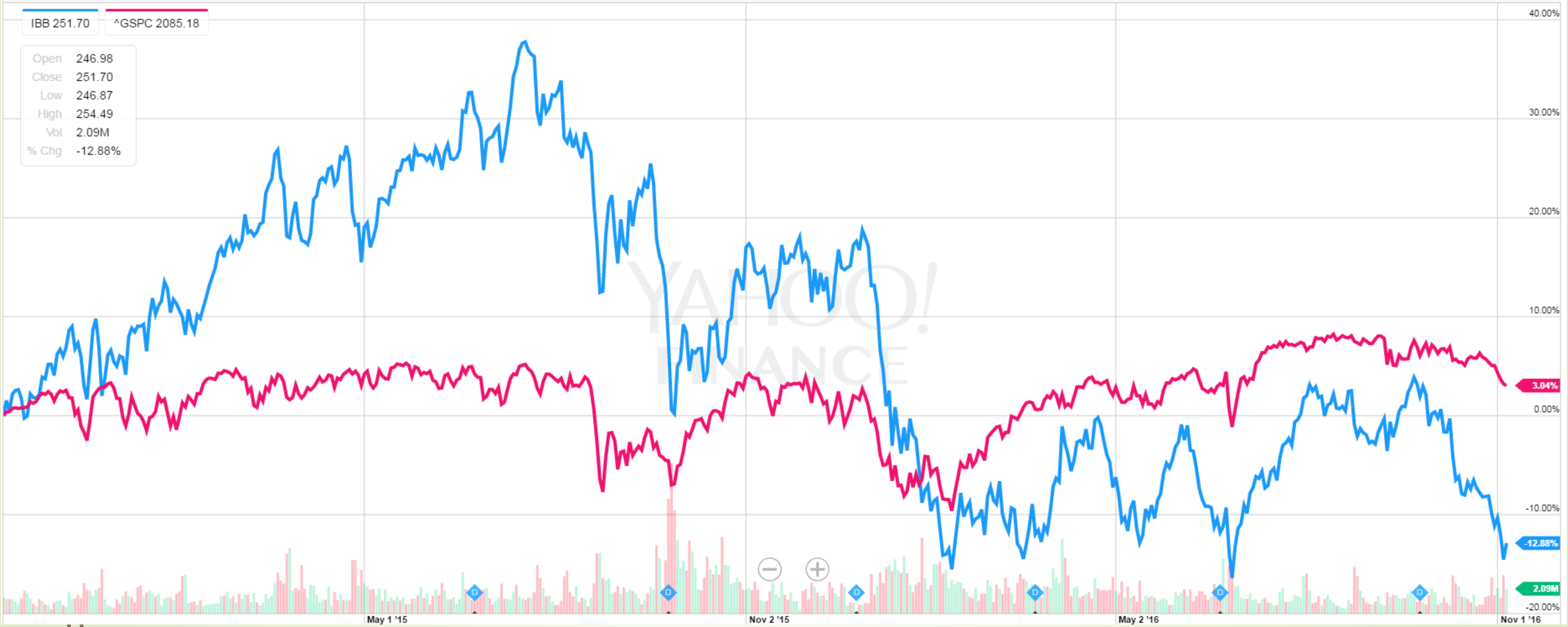
USA: Investing in Tech

- XLK



USA: Investing in Biotech

- IBB

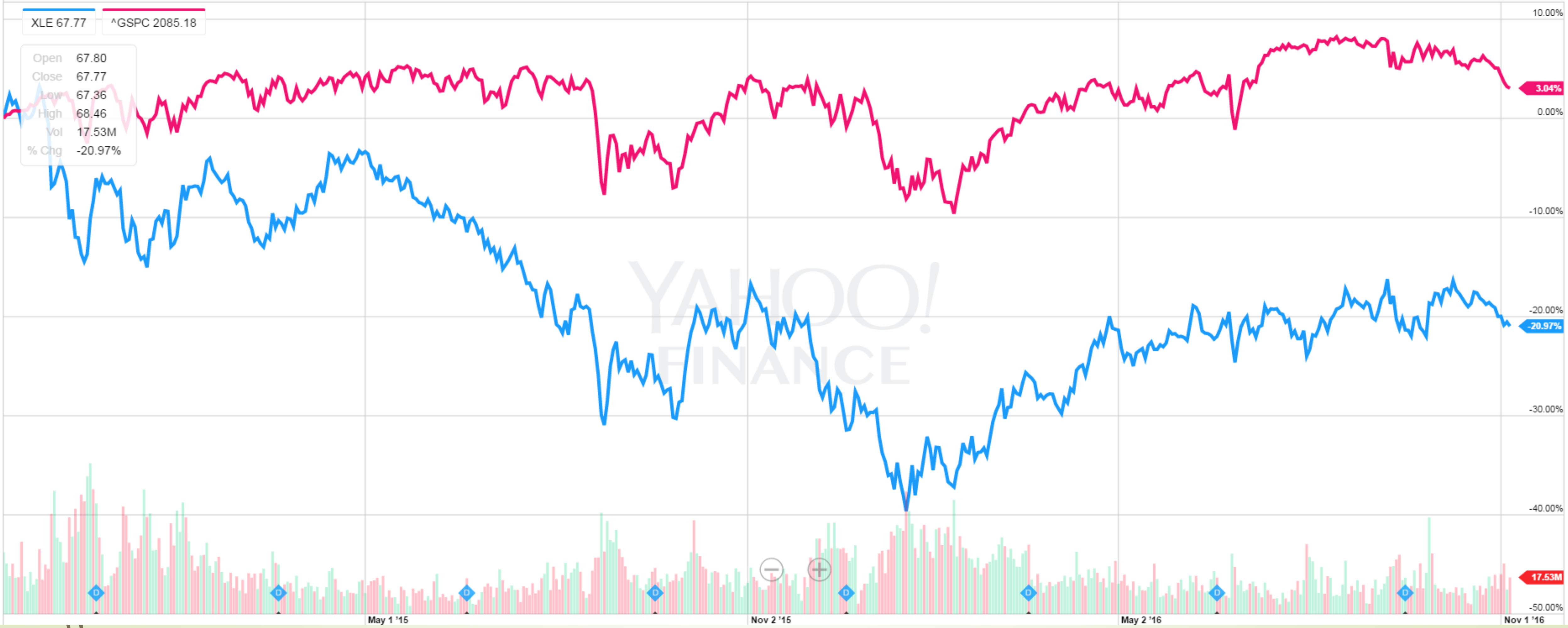


USA: Investing in Semiconductors

- SMH

USA: Investing in Energy

- XLE

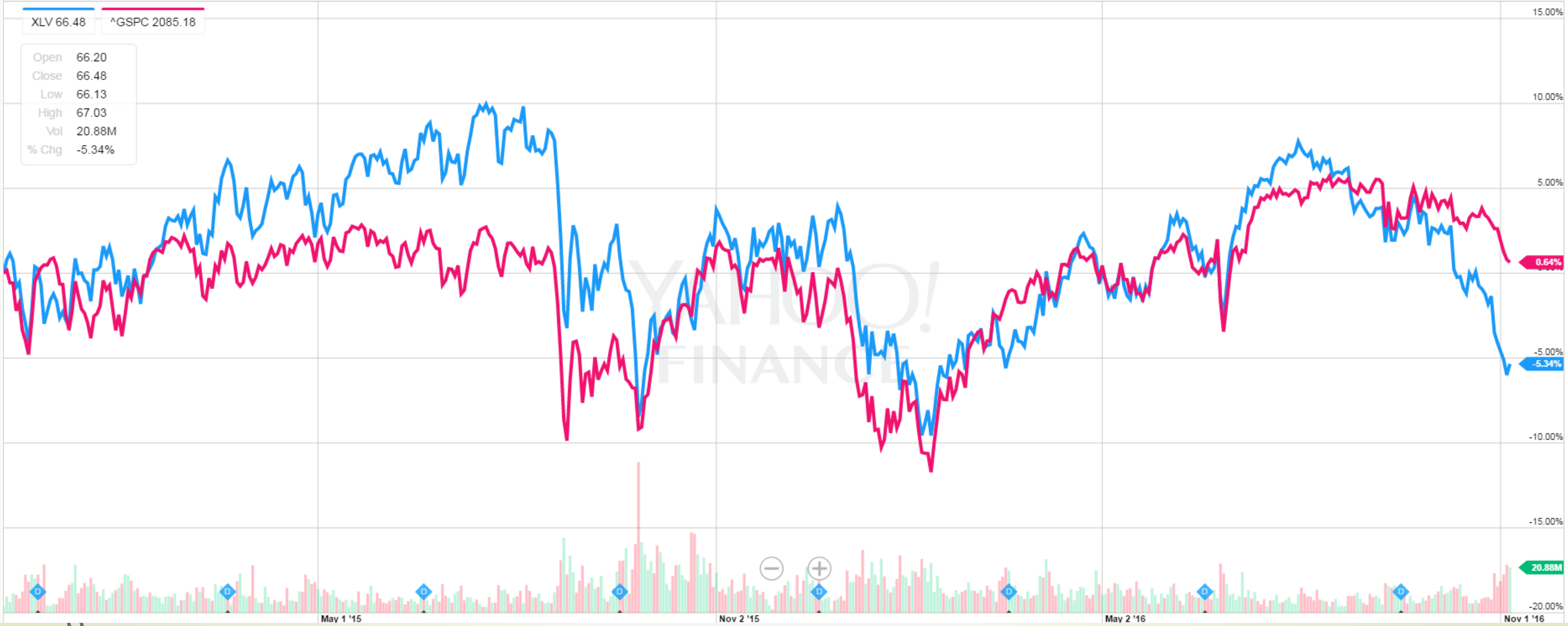


USA: Investing in Green Energy

- TAN

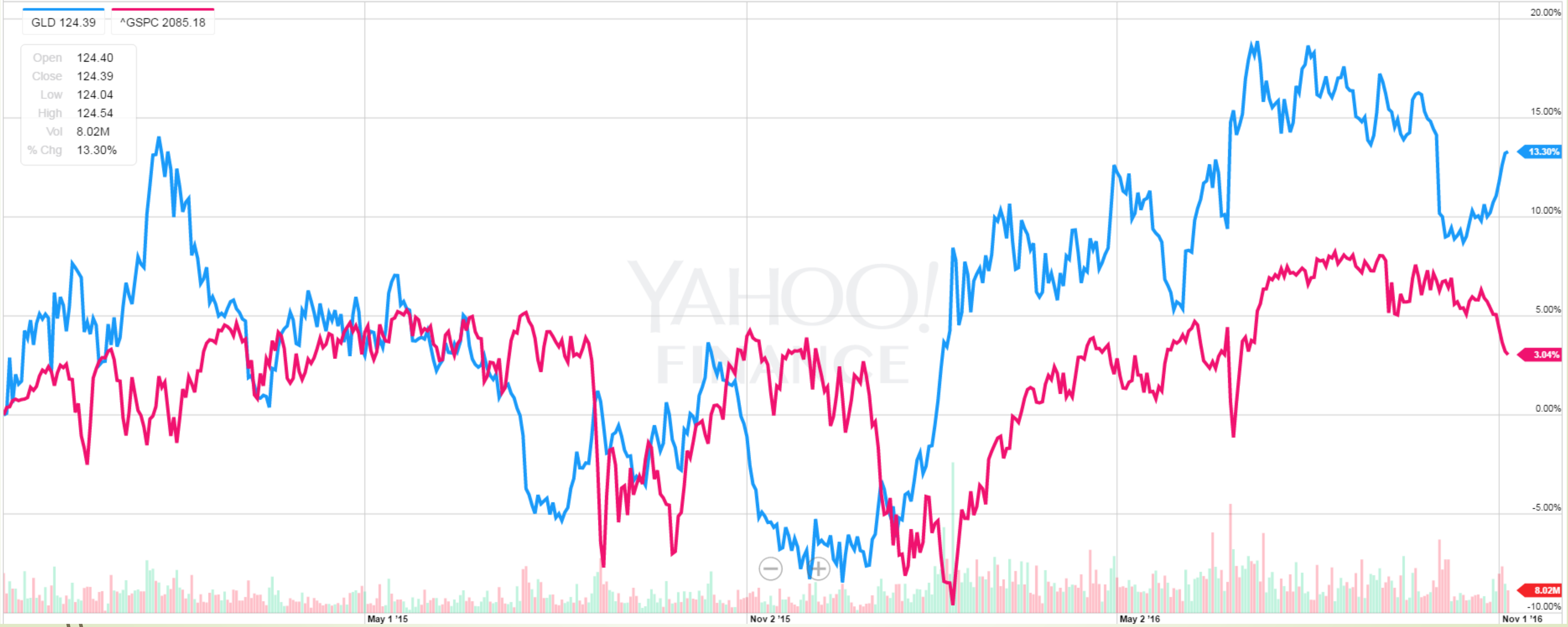
USA: Investing in Healthcare

- XLV



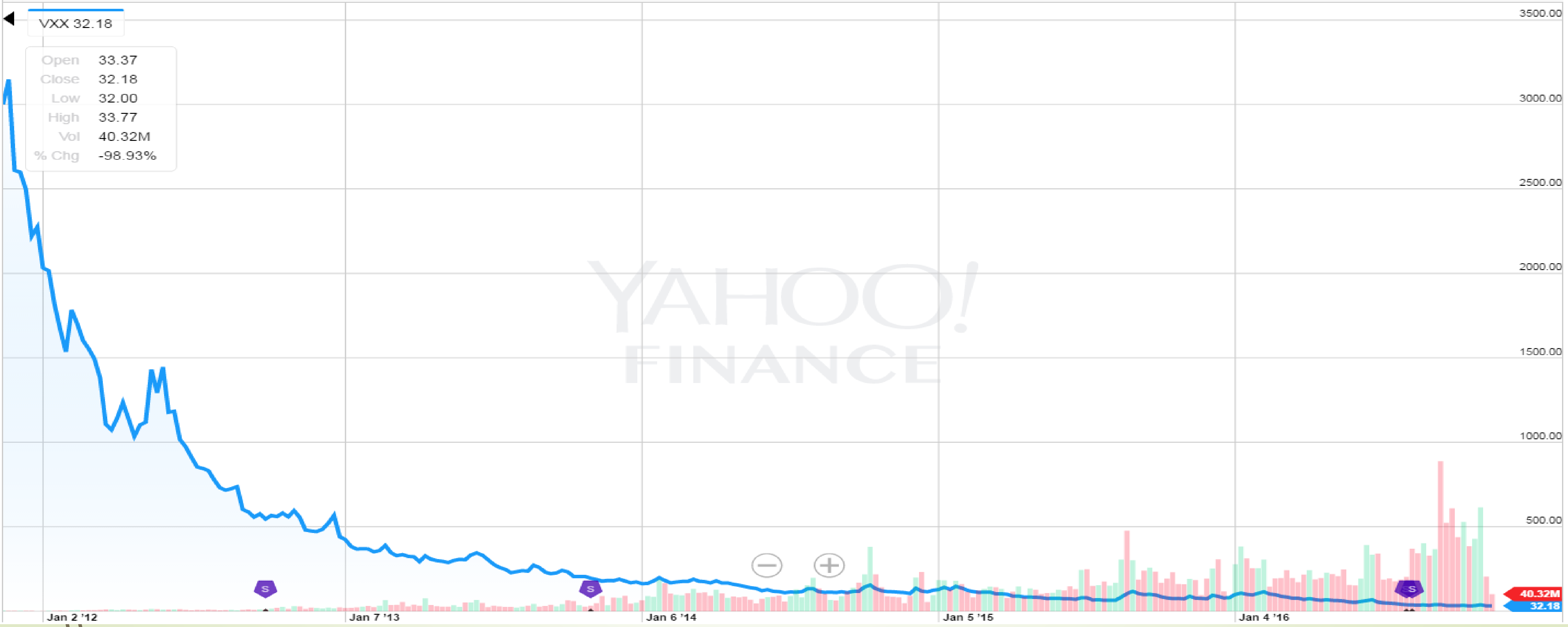
USA: Investing in Precious Metals

- GLD

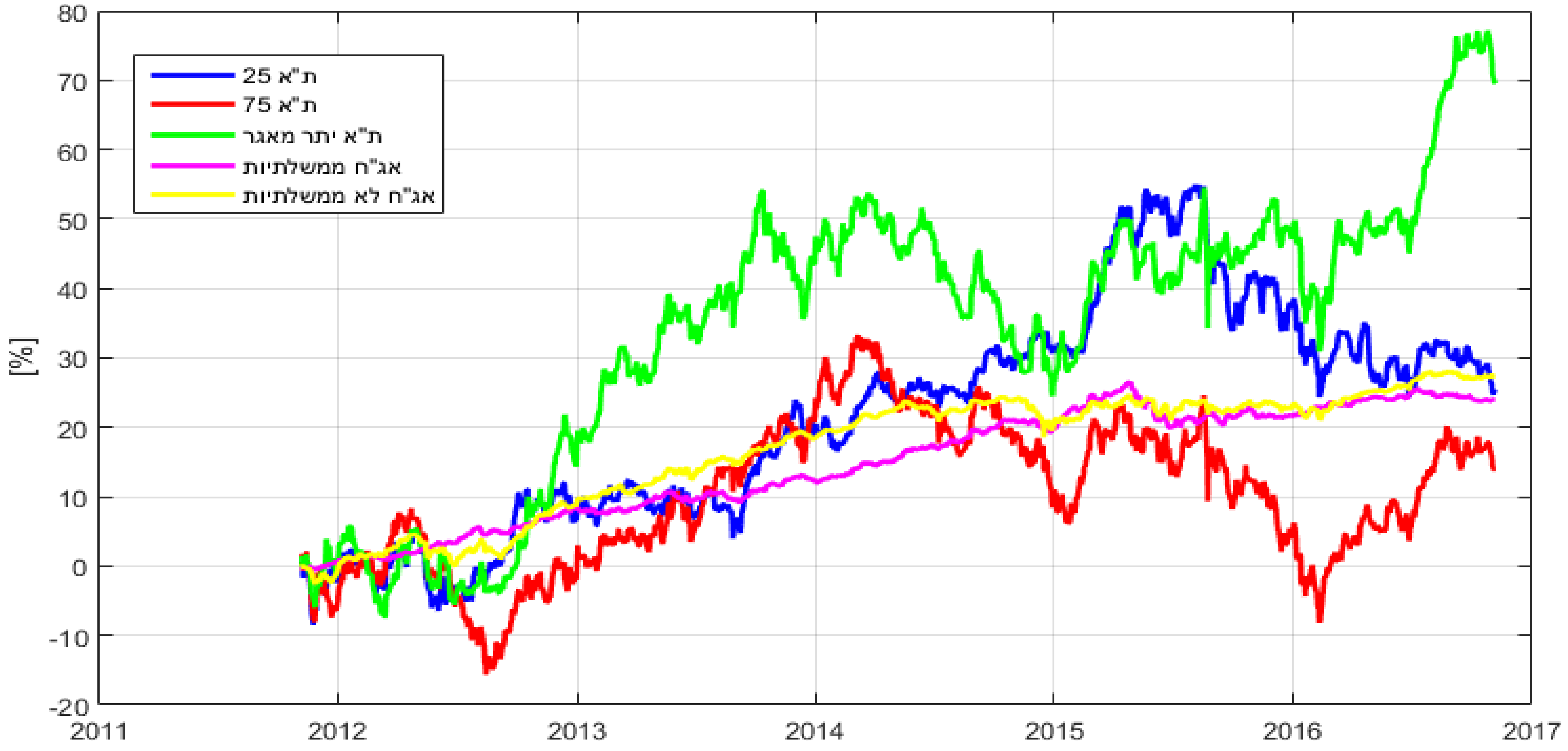


Investing in Volatility? Not really and be careful here!

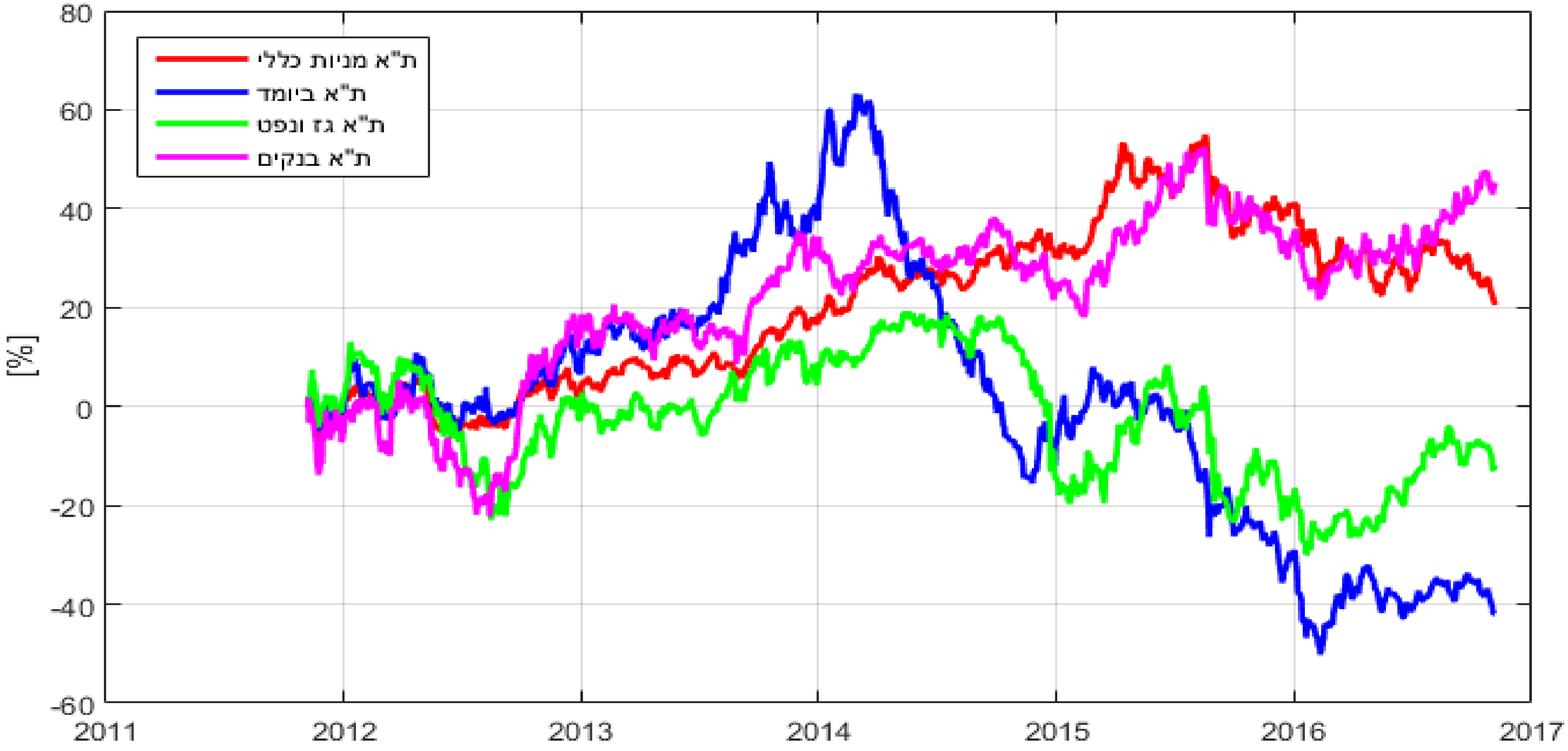
- VXX



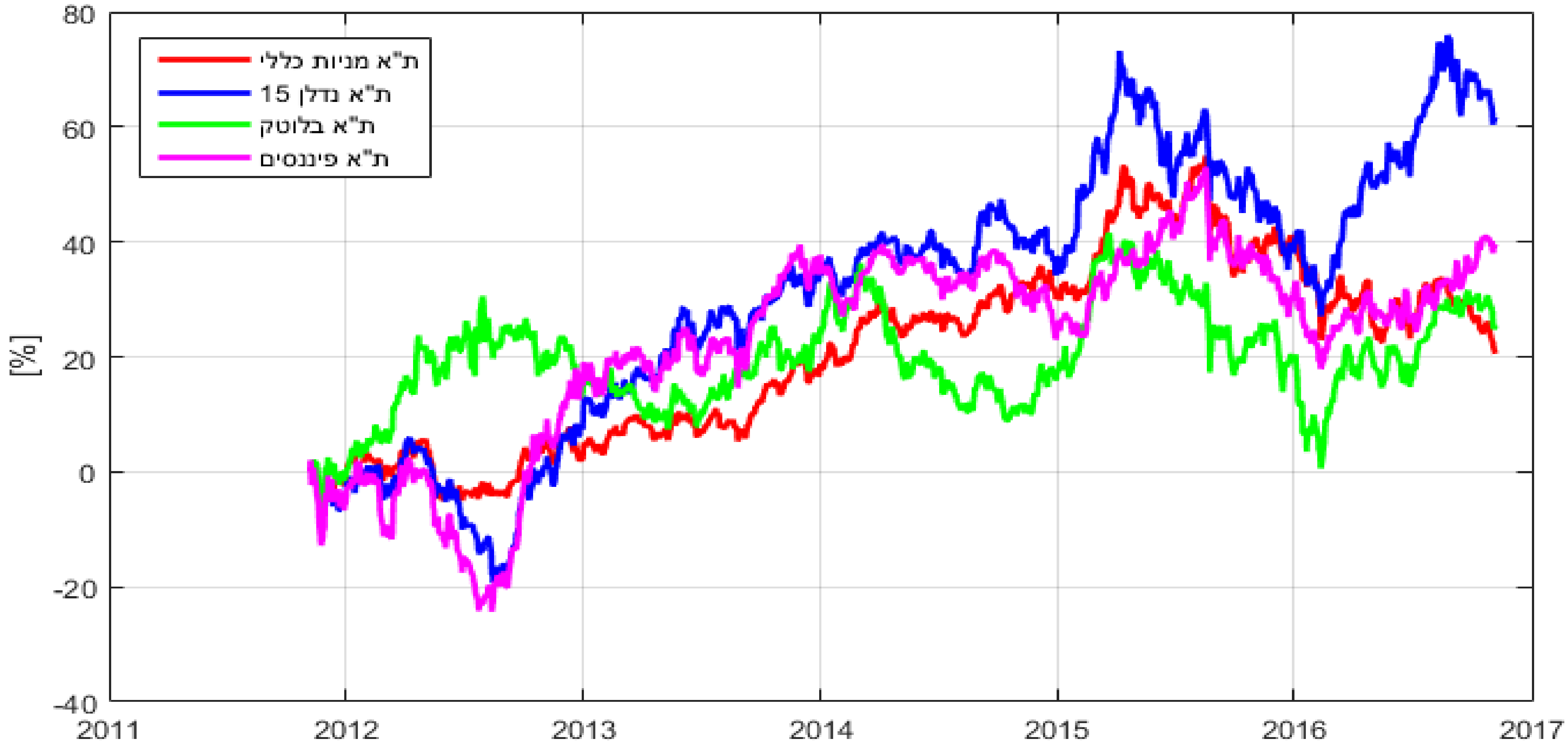
Back Home: Asset Allocation/Security Selection



Israel: Sector Investing



Israel: Sector Investing



Some Calculations: Rate of Return for a Single Period Investment

In what follows, we implement several return computations, starting with a simple single-period example.

Suppose you invested $1,000 in AIG's stock at $25 per share. After one year, the stock price increases to $35.

For each AIG stock, you also receive $1 cash dividend during the year.

a. How many shares did you buy?
b. What is your investment rate of return in % and in $?

Calculating Investment Return: Dividend Yield vs. Capital Gain

- Obviously, you bought 40 stocks.
- Dividend Yield = $1/$25 = 4%
- Capital Gain = ($35 - $25)/$25 = 40%
- Total Percentage Return = 4% + 40% = 44%
- Total Dollar Return = 44% of $1,000 = $440
- At the end of the year, the value of your $1,000 investment becomes $1,440, from which $1,400 is the value of the 40 AIG stocks and $40 is cash due to dividend payment.

What if Dividends are Invested?

Suppose now that AIG paid the dividend in the beginning of the year and you invested the dividend in a riskfree cash account delivering 2% annual return. What is the investment return?

- Capital gain – 40% (does not change)
- Invested dividend – $4\% \times 1.02 = 4.08\%$
- Total return – 44.08%

Stock Split

On January 1, you buy 10 stocks of Google. The stock price is $800. In the middle of the year Google announces a two-to-one split. That is, shareholders hold twice as many stocks. In the end of the year Google pays $5 per share dividend. The ex dividend share price is $402 (the cum dividend price is $407).

a. Why would a company initiate a split?
b. What is the annual return on holding Google Share of stock?
c. What is the long run performance following stock split?

Buying on Margin, Initial Margin, and Maintenance Margin

On January 1st, 2013, you use your entire cash balance $10,000 in addition to a $10,000 loan made by your bank to buy stocks. In particular, you purchase 400 shares of Hain Celestial Group Inc. (HAIN). You pay 5% interest on the loan. On June 30th, you get a cash dividend of $1 per share. You deposit the dividend amount in a risk-free account delivering 1% per six months. On December 31, 2013, HAIN stock price is $75.

a. What is the net investment return?

b. What is the HAIN stock price below which you get a margin call assuming that the maintenance margin is 30% (the initial margin is 50%)?

Buying on Margin – the Case of BCOM

- B-communication (BCOM) is a public company traded in both TASE as well as NASDAQ.
- BCOM establishes a good practical example of buying on margin.
- It purchased the control on BEZEQ using its own equity as well as loans from both banks and other institutions.
- It services the loans through dividend payments from BEZEQ.
- Due to its levered position, BCOM has higher expected return but also higher risk relative to BEZEQ.
- In a sense, BCOM is a sort of long run call option on BEZEQ stock.

Buying on Margin – Use Levered ETFs

- Nowadays, you can "sort of" buy on margin using levered ETFs.
- Here are a few examples.
  - SPXL delivers three times ***daily*** returns on the S&P500.
  - YINN delivers three times ***daily*** returns on several large Chinese stocks traded in Hong Kong.
  - BIB delivers two times ***daily*** returns on a biotech index.
  - There are levered ETFs on economies, sectors, commodities, bonds, etc.
- One major difference between levered ETFs and buying on margin is the daily rebalancing featuring levered ETFs.
- Due to daily rebalancing, it can be proven that when the holding period is long enough the value of a levered ETF goes to zero even when the underlying asset valuation goes north. This is the volatility drag.

Short Sale

- A market transaction in which an investor sells securities he/she does not own in anticipation of a price decline and is required to return an equal number of shares at some point in the future.

- A short seller would make money if the stock price falls, while a long position makes money when the stock price goes up.

- ***Long position: buy low sell high.***
- ***Short position: sell high buy low.***

Short Sale: Example

- Suppose 1,000 shares are short sold by an investor at $25 apiece.
- Suppose the shares fall to $20 and the investor closes out the position.
- To close out the position, the investor will need to purchase 1,000 shares at $20 each ($20,000).
- The investor earns $5,000.
- What is the % return? To answer - you have to know the initial net investment (equity) – you need to take account of initial margin.

Short Sale: Margin

- There are margin requirements for a short sale in which 150% of the value of the shares shorted needs to be initially held in the account.
- Therefore, if the value of the short position is $25,000, the initial margin requirement is $37,500 (including the $25,000 of proceeds from the short sale).
- This prevents the proceeds from the sale from being used to purchase other shares before the borrowed shares are returned.

Short Sale: Margin

- Short selling is an advanced trading strategy with many unique risks and pitfalls.
- Investors are advised to be extra cautious about short selling because this strategy involves unlimited losses.
- A share price can only fall to zero, but there is no upper bound.
- The transaction is typically expansive and there are other impediments associated with short selling.

Short Sale : The case of Herbalife

- **Herbalife (NYSE: HLF)** is a nutrition company that sells products through a network of individuals.
- Bill Ackman, ***once*** perceived a distinguished hedge fund manager, started publicly bashing the stock.
- His fund, Pershing Square, began short-selling HLF shares in May 2012.
- Though Ackman did not disclose his average sale price, HLF shares were trading around $45 when he started short-selling the stock.
- What we do know is that Pershing Square sold short around one billion worth of HLF outstanding shares.
- In December 2012, Ackman went public with his position, sharing his view through the media that HLF was an unsustainable pyramid scheme.

The case of Herbalife

- HLF shares dropped to the mid-20s.
- But shorting a company is risky, among others due to short squeeze.
- Short squeeze can occur in a heavily shorted stock. If there is increased buying of the stock, the share price rises. But with a large number of shares tied up by short sellers, the price can rise very quickly.
- The first hedge fund manager to bet against Ackman was Daniel Loeb.
- Carl Icahn expressed his own pro HLF outlook by purchasing a large portion of the company.
- Later, George Soros acknowledged his own long position in HLF.
- At that point Ackman closed his short position and instead bought long term put options on HLF (he was subject to a short squeeze).
- Long story short – the regulator decided in 2016 that HLF was not a scheme.
- Ackman lost big time.

Other case - Valeant

- **Valeant Pharmaceuticals International (NYSE: VRX)** is a specialty drugs company based in Canada.
- Between March 2015 to February 2016 the same Bill Ackman acquired a long position on Valeant: buying stocks and call options and selling put options.
- It was Pershing second largest equity position, quite a huge position
- In 2016, the short seller firm CITRON published a report claiming that Valeant is the Pharmaceutical Enron.
- In February 2016, Valeant disclosed that it was under investigation by the U.S. Securities and Exchange Commission.
- The stock price slides from above $250 to around $20.
- Ackman and his hedge fund recorded massive losses.

Would you shell short…??

- Would you sell short TSLA, NFLX, AMZN, or German Bonds?
- Not so fast!!
- While their prices seem to be too high relative to fundamentals (it has been so for several years), the common wisdom is that you should be careful going against the trend.
- Technicians say: **the trend is your friend**.
- Indeed, reconsider if you attempt to sell short because “the price seems too high” – it is not highly suggested to argue with the crowd.
- You do not have to run with the crowd – but be careful running against it.

Short Selling – Use Inverse ETFs

- Nowadays, you can short sell via purchasing inverse ETFs.
- Here are a few examples.
  - DUST takes a short position on gold.
  - TMV takes a short position on long term US bonds.
  - ERY takes a short position on energy stocks.
  - PSQ takes a short position on NASDAQ stocks.
- There are many other examples.
- Notice that also here rebalancing is made on a daily basis.
- Over the very long run, the inverse ETF value approaches zero.
- In general, levered and inverse ETFs are short term instruments.

Multi-Period Rate of Return

- The previous examples compute a single period (e.g., one day, one month, or one year) rate of return.

- Suppose your investment spans three periods, e.g., three months or three years.

- The corresponding periodical returns are $R_1, R_2,$ and R_3.

- The total three-period holding period return (HPR) is

$$HPR = (1 + R_1)(1 + R_2)(1 + R_3) - 1$$

Multi-period Return: Example

Let $R_1 = 10\%, R_2 = 5\%,$ and $R_3 = 7\%$.

$$HPR = (1 + 0.1)(1 + 0.05)(1 + 0.07) - 1 = 23.59\%$$

So if you start with $100,000 - after three years you have $123,590.

A common mistake is to compute HPR=10+5+7=22%.

As Albert Einstein points out: There is no greater force than compounded interest.

Annualizing Returns

- Often times, you want to compare various investments, each of which applies to a different time period.

- Then you should express returns on a per-year, or *annualized*, basis.

- Such a return is often called an effective annual return (EAR).

$$EAR = (1 + holding\ period\ percentage\ return)^m - 1$$

where m is the number of holding periods in a year.

Annualizing Returns: Example

- You buy GILD for 34 and sell it 3 months later for \$38.
- There were no dividend payments.
- What is your holding period percentage return and your EAR?

$$HPR = \frac{38-34}{34} = \frac{4}{34} = 0.117647 = 11.7647\%$$

$$EAR = \ (1 + Holding\ Period\ Percentage\ Return)^m - 1$$
$$= (1 + 0.117647)^4 - 1 = 56\%$$

Annualizing Returns: Example

- What if the holding period corresponds to six months (m=2)?

$$EAR = (1 + Holding\ Period\ Percentage\ Return)^m - 1$$
$$= (1 + 0.117647)^2 - 1 = 24.9\%$$

- What if the holding period amounts to two years ($m = \frac{1}{2}$)?

$$EAR = (1 + Holding\ Period\ Percentage\ Return)^m - 1$$
$$= (1 + 0.117647)^{1/2} - 1 = 5.7\%$$

Historical Average Return and Volatility

- Two useful statistics that help us summarize historical asset return data is the average return and the standard deviation (STD).
- Average return stands for investment's reward.
- STD characterizes the investment's risk.
- The risk-return tradeoff: higher risk should be compensated by higher return.
- The risk-return tradeoff could be violated in the data (more later).
- There are higher moments of asset returns – skewness and kurtosis – that could affect investment decisions.

Estimating Average Return and Volatility: Example

The spreadsheet below shows us how to calculate the average return, the variance, and the standard deviation

(1) Year	(2) Return	(3) Average return	(4) Difference: (2)-(3)	(5) Squared: (4) × (4)
1926	13.75	12.12	1.63	2.66
1927	35.70	12.12	23.58	556.02
1928	45.08	12.12	32.96	1086.36
1929	-8.80	12.12	-20.92	437.65
1930	-25.13	12.12	-37.25	1387.56
Sum:	**60.60**		**Sum:**	**3470.24**
Average:	**12.12**		**Variance:**	**867.56**
			Standard Deviation:	**29.45**

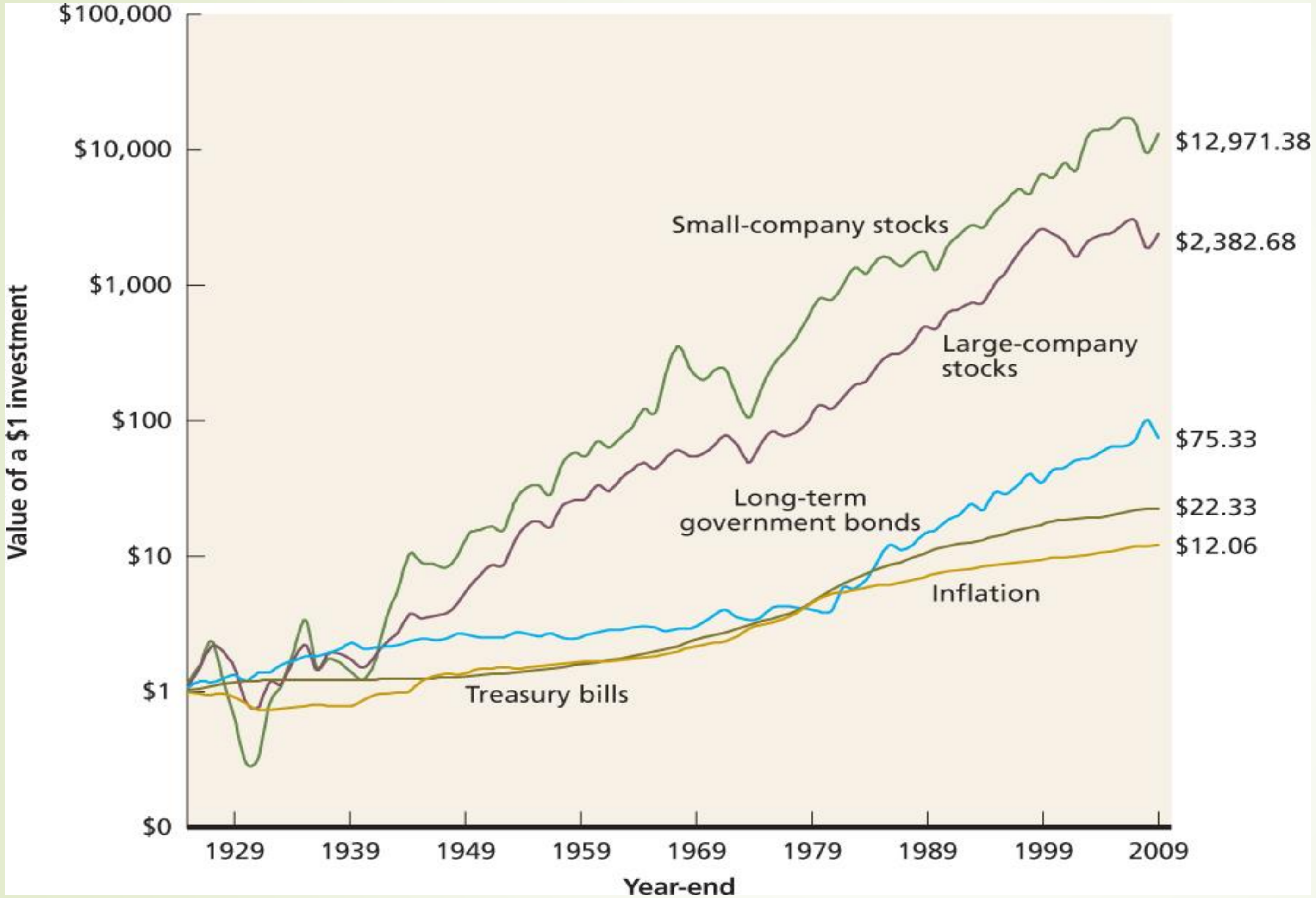
Equity, Bond, or Cash? The Empirical Evidence

- We next present an extensive evidence about long run investment payoffs corresponding to investments in stocks, bonds, and cash (bills).

- You will find out that stocks are much more profitable but also much more risky.

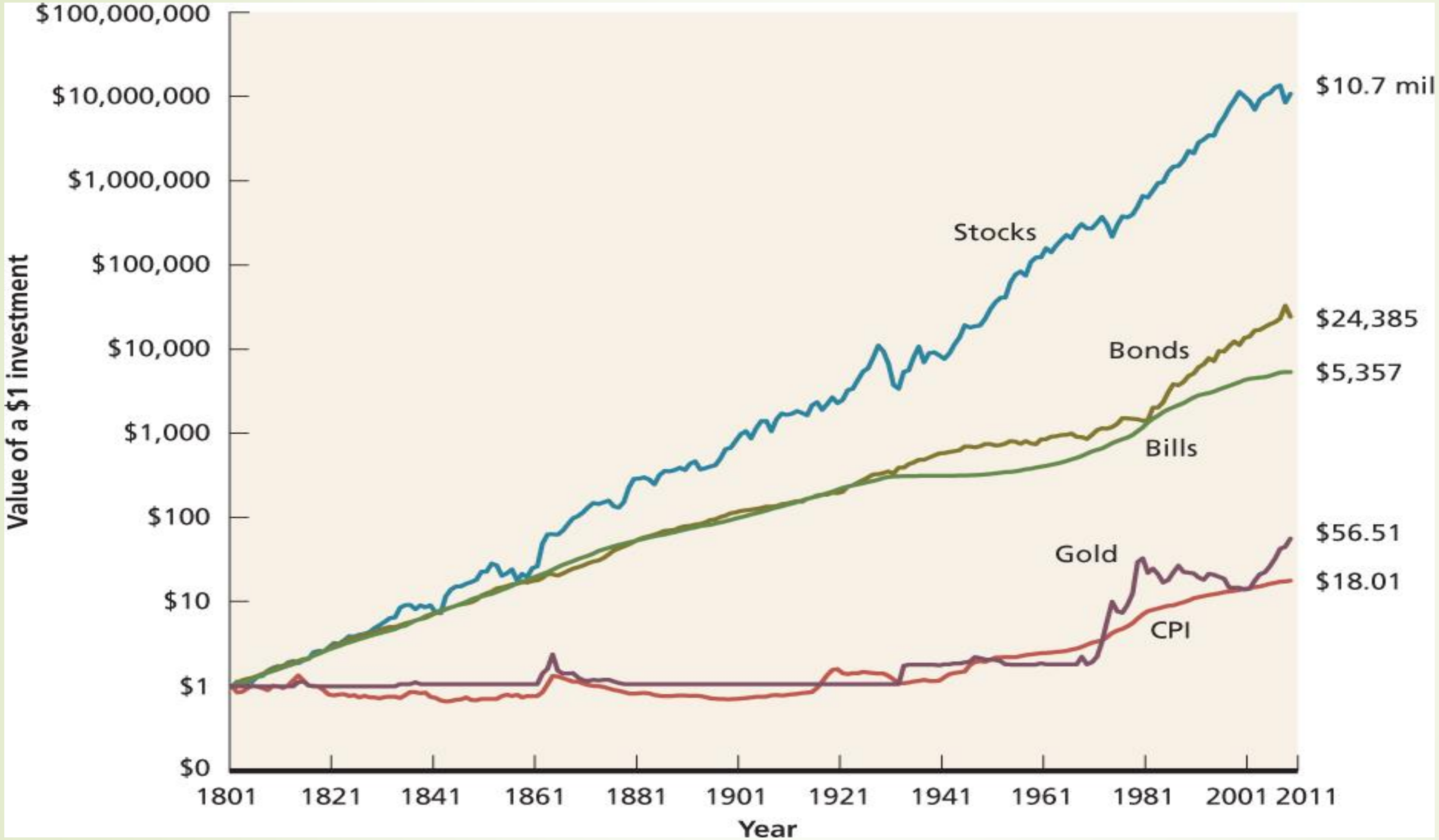
- All investment instruments have been able to hedge against inflation risk over the long investment horizon.

A $1 Investment in Different Types of Portfolios, 1926—2009



Source: *Global Financial Data* (www.globalfinancialdata.com) and Professor Kenneth R. French, Dartmouth College. Notice that the vertical scaling is logarithmic.

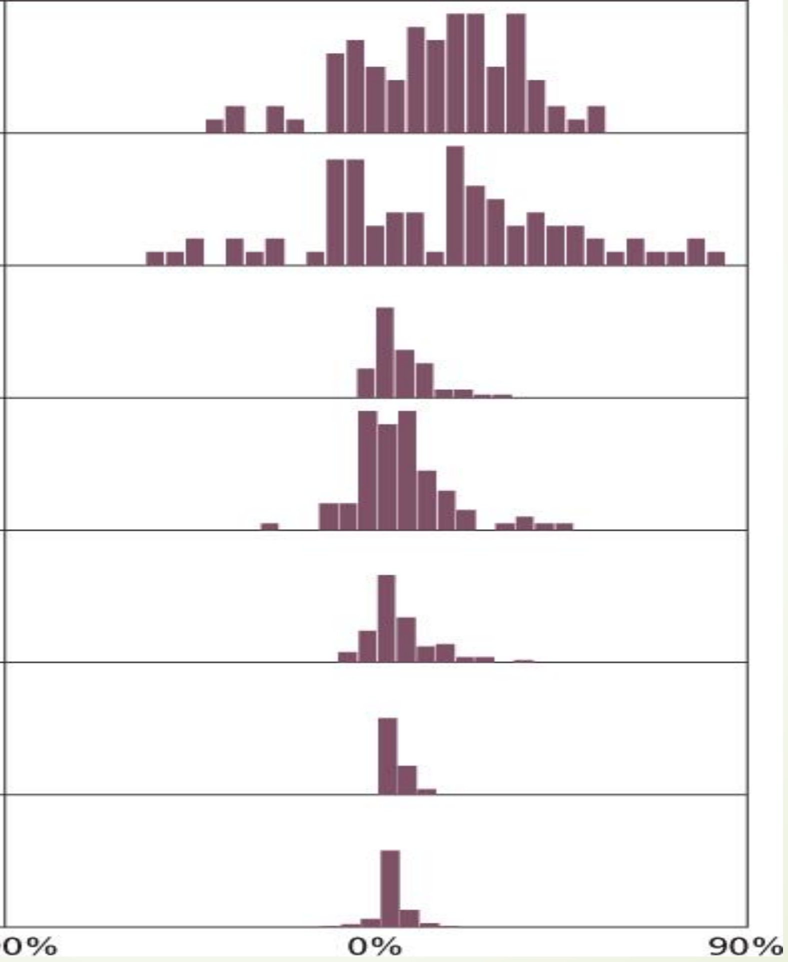
Financial Market History: A Longer Horizon



Source: Jeremy J. Siegel, *Stocks for the Long Run*, 3rd ed. (New York: McGraw-Hill, 2003). Update through 2009 provided by Jeremy J. Siegel. *Global Financial Data* (www.globalfinancialdata.com) and Professor Kenneth R. French, Dartmouth College

Historical Returns, Standard Deviations, and Frequency Distributions: 1926—2009

Series	Average return	Standard deviation	Frequency Distribution
Large-company stocks	11.7%	20.5%	
Small-company stocks*	17.7	37.1	
Long-term corporate bonds	6.5	7.0	
Long-term U.S. government bonds	5.9	11.9	
Intermediate-term U.S. government bonds	5.6	8.1	
U.S. Treasury bills	3.8	3.1	
Inflation	3.1	4.2	



What is Risk Premium?

Risk premium (RP) is the average return of an investment over and above the risk free rate (proxied by T-bill rate).

Investment	Average Return (%)	Risk Premium (%)
Large stocks	11.7	7.9
Small stocks	17.7	13.9
Long-term corporate bonds	6.5	2.7
Long-term government bonds	5.9	2.1
U.S treasury bills	3.8	0.0

What is the Sharpe Ratio?

The Sharpe ratio – a common investment performance measure – divides the RP by the investment's volatility. It represents the price of risk.

Series	RP (%)	Vol (%)	SR
Large-company stocks	7.9	20.5	0.39
Small-company stocks	13.9	37.1	0.38
Long-term corporate bonds	2.7	7.0	0.39
Long-term government bonds	2.1	11.9	0.18

Geometric vs. Arithmetic Average

- Returns over three periods are 20%, 0%, and -10%.

- Arithmetic Average (AA)

$$\frac{0.20\ +\ 0\ +\ (-0.1)}{3} = 3.33\%$$

- Geometric Average (GA)

$$[(1+0.2)(1+0)(1-0.1)]^{\frac{1}{3}} - 1 = 2.60\%$$

GA vs. AA: The Evidence

Series	GA (%)	AA (%)	Standard Deviation (%)
Large-company stocks	9.7	11.7	20.5
Small-company stocks	11.9	17.7	37.1
Long-term corporate bonds	6.3	6.5	7.0
Long-term government bonds	5.3	5.9	11.9
Intermediate-term government bonds	5.3	5.6	8.1
U.S treasury bills	3.7	3.8	3.1
Inflation	3.0	3.1	4.2

Geometric vs. Arithmetic Average

- If you want to assess ex post performance – geometric average establishes the essential informative measure.
- Ex ante, arithmetic average could make some sense.
- Most investment returns are reported as arithmetic average since that is the best performance that can be reported.
- Eventually, volatility lowers investment returns, as displayed in the previous figures.
- Perhaps the other side of the coin: Suppose you start investing 10 years ago with initial investment of $1,000,000. Nowadays the value is $1,560,000. You did not withdraw cash (e.g., dividends) from the account.
- Then it is easy to compute the holding period return as well as the geometric average.
- But the figures do not reveal the arithmetic average.
- For a given holding period return – the arithmetic average gets larger with higher volatility.

An International Perspective:
1990-2009 vs. Longer Periods

Equities	1990-2009 Geometric Mean	Sharpe Ratio	Long History Geometric Mean	Long History Sharpe Ratio	Starting Date
Countries/Regions (in $)	(nominal)		(nominal)		
World	5.9	0.23	8.6	0.35	1900
U.S.	8.4	0.32	9.3	0.37	1900
Germany	6.6	0.23	6.4	0.18	1900
U.K.	8.2	0.32	8.4	0.29	1900
Japan	-2.6	-0.10	7.4	0.28	1900
North America	8.4	0.38	9.8	0.36	1970
Europe	8.3	0.35	10.8	0.39	1970
Pacific	0.1	0.20	9.8	0.27	1970
Emg LatAm	18.4	0.64	22.2	0.80	1988
Emg Asia	5.3	0.20	10.0	0.31	1988

Indexes – Why are they Useful?

- Track average returns.
- Benchmarks for performance of active management (e.g., mutual funds).
- Base for derivatives.
- Base of ETFs and mutual funds, both of which achieve investment diversification quite easily and cheaply.

Market Wide Equity Indexes

- S&P 100,500 (US)
- NASDAQ (US)
- The Shanghai Composite Index (China)
- Nikkei 225 (Japan)
- FTSE (Financial Times of London)
- DAX (Germany)
- Hang Seng (HKG)
- TSX (CANDA)
- Tel Aviv 100 (Israel)

Bond Indexes

- Barclays Capital Aggregate Bond Index
- Salomon Smith Barney World Government Bond Index
- JP Morgan Emerging Markets Bond Index
- Merrill Lynch High Yield Master
- Telbond 20, 40, 60

Construction of Indexes

How are assets weighted in an index?

- Price weighted (DJIA).

- Market-value weighted (S&P500, NASDAQ).

Constructing Indexes: A Three-Security Example

	JPM	F	KO
Number of Outstanding Stocks	100	500	200
Price as of Oct. 12	15	5	10
Price as of Nov. 12	18	5	9
Market Value as of Oct. 12	1500 (1/4)	2500 (5/12)	2000 (1/3)

Constructing the Dow Jones Index: An Example

Weight prices:

- Dow Jones level in Oct 12: (15+5+10)/3 = 10
- The level in Nov 12: (18+5+9)/3 = 10.67
- Monthly return on the Dow Jones index is 10.67/10-1 = 6.7%

Is it Really a Price Weighted Index?

The prices of JPM, F, and KO are 15, 5, and 10, which means that the corresponding weights in the DJ index are 1/2, 1/6, and 1/3.

- Thus, the price weighted average is

$$1/2\times 20\% + 1/6\times 0\% + 1/3\times(-10\%) = 6.7\%.$$

- So we can compute the return on a price weighted index in the two ways suggested here.

The Dow Jones: What if?

- What if stocks are dropped from and/or added to the DJ Index?
- What if there are stock splits?
- What if there is a cash dividend payment?
- The first question above will be answered here. The second and third – I'd leave both as an home assignment.

Changing the Divisor – An Example

- Day 1 of index:

Company	Price
GM	40.56
Nordstrom	25.91
Lowe's	53.68
Sum:	120.15
Index:	40.05

- Divisor = 3

- Before Day 2 starts, you want to replace Lowe's with Home Depot, selling at $32.90.
- To keep the value of the index the same, i.e. 40.05

Company	Price
GM	40.56
Nordstrom	25.91
Home Depot	32.90
Sum:	99.37

- 99.37 / Divisor = 40.05
- Divisor is: 2.481

Moving to Value Weighted Index such as the S&P

- The monthly returns on JPM, F, and KO are 20%, 0%, and -10%, respectively.

- The value weighted return (S&P and NASDAQ) is computed as:

$$1/4\times 20\% + 5/12\times 0\% + 1/3\times(-10\%) = 1.67\%$$

- Why are the price weighted and value weighted averages so different?

Computing the S&P Index Level: An Example

- If the S&P index level in Oct. 12 is 1000, then the index level in Nov. 12 would be $1000\times(1+1.67\%)=1016.7$.

- What if at least one of those three stocks pays a dividend? For instance, assume that the 1.67% total return consists of 1% capital gain yield and 0.67% dividend yield – what is the S&P index level in Nov 12?

The index is compounded only by the capital gain component. In Israel, the MAOF compounds both the capital gain and dividend.

On the DJ Components

- There are 30 stocks establishing the Dow Jones Industrial Index
- The DJ membership is determined by a special committee.
- AAPL the largest market cap stock only recently (March 19, 2015) entered the DJ. AT&T exited.
- The other most substantial changes occurred in September 2013.
  - BAC, AA, and HP exit.
  - GS, NIKE, and VISA enter.
- The previous major change goes back to 2004.

Appendix I: Advice from the Wizard – The Warren Buffett's Rules for Investing

- The next chapter deals with market efficiency and potentially profitable trading strategies.
- As a quick promo to the upcoming chapter let us quickly summarize some of Warren Buffett's rules for investments.

- Rule No.1: Never lose money.
- Rule No.2: Never forget rule No.1.

Warren Buffett – Value Investing

It is far better to buy a wonderful company at a fair price than a fair company at a wonderful price.

- The basic premise of Buffett's investing style is buying something for less than it is actually worth.
- This sounds simple, but unearthing these stocks and prove difficult and it's easy to mistake a company that is unloved by the market because nobody has spotted its opportunity with one that is simply a dog.
- For that reason, Buffett applies some of the measures that are listed in the following slides.

Strong Profitability

If a business does well, the stock eventually follows.

- Buffett prefers to invest in companies with a proven level of strong profitability, giving more credence to this than what analysts predict will happen in the future.
- He looks at a number of measures to assess a business's profitability, including return on equity (ROE), return on invested capital (ROIC), and a company's profit margin.

ROE

- Buffett uses ROE to examine how well a company performs relative to other businesses operating in the same sector.
- You can calculate the ROE by dividing the company's net income by the shareholder's equity. It is believed that Buffett prefers a company that has an ROE in excess of 15%.
- He also looks for companies with above average profit margins, which can be calculated by dividing net income by net sales. The higher the ratio, the more profitable the company based on its level of sales.

Not too much Debt

- A company with a high ROE could be fuelled by substantial levels of debt, which Buffett is keen to avoid.
- Buffett doesn't like over-indebted companies, as he says each year in his Berkshire Hathaway letters, because such companies could become vulnerable during a credit squeeze or when interest rates are rising, as they have been doing recently.

Warren Buffett – Term of Investment

- Five to ten years isn't ideal, it is the starting point.

- "When we own portions of outstanding businesses with outstanding managements, our favorite holding period is forever."

Appendix II: Biggest Daily Price Appreciation and Depreciation since 1980 - S&P 500

Date	Change		Date
19/10/1987	**-20.47%**	**10.79%**	**28/10/2008**
15/10/2008	-8.72%	9.93%	13/10/2008
29/09/2008	-8.49%	9.10%	21/10/1987
26/10/1987	-8.27%	6.82%	13/11/2008
01/12/2008	-8.15%	6.55%	23/03/2009

(SOURCE: YAHOO FINANCE)

Appendix III: Largest Corporate America Annual Earnings of All Time

Corporate	Year	USD real earning (Bn $)
Fannie Mae	**2013**	**84**
ExxonMobile	2008	48.55
Apple	2012	41.73
Ford Motor Company	1998	30.39
Citigroup	2005	28.2

(SOURCE: WIKIPEDIA)

Appendix IV: Largest Corporate America Annual Losses of All Time

Corporate	Year	USD real loss (Bn $)
AOL Time Warner	2002	123.16
AIG	2008	106.62
Fannie Mae	2009	77.77
JDS Uniphase	2001	71.66
Freddie Mac	2008	54.54

(SOURCE: WIKIPEDIA)

Appendix V: Bulls vs. Bears

- A bull fights by striking UP with his horns. A bull is a buyer – an investor who bets on a rally and profits from price increase.

- A bear flights by striking DOWN with his paws. A bear is a seller –an investor who bets on a decline and profits from falling prices.

End-of-Chapter Questions

- In the end of this chapter as well as follow-up chapters there are several questions about the material we covered.
- A few questions are taken from past year exams – so make sure you get them all in letter and spirit.
- Due dates for submitting the problem sets will be announced in class.
- Submit your assignment directly to the TA via email.
- Problem sets should be solved on an individual basis.

Questions

- **Q1**: Which of the following is true of the S&P500 index?
  a. It is a value-weighted average of 500 stocks.
  b. It is a price-weighted average of 500 stocks.
  c. The divisor must be adjusted for stock splits and cash dividends.
  d. It is an equal-weighted average of 500 stocks.

- **Q2**: The Value Line Index is a geometric average of the return of about 1,700 firms. Following that weighting method, what is the return on the index based on three stocks only with rates of return 10%, -7%, and 6%?

Questions

- **Q3**: You have been given this probability distribution for XYZ stock:

State of Economy	Probability	HPR
Boom	30%	8%
Normal Growth	50%	4%
Recession	20%	-5%

What is the expected return and volatility for XYZ stock?

- **Q4**: The geometric average for the six year period 2006-2011 is 10% per year. The geometric average for the two year period 2006-2007 is 11%. The geometric average for the two year period 2008-2009 is 8%. What is the geometric average for the two year period 2010-2011?

Questions

- **Q5**: You purchase a share of Google stock for \$820. One year later, after receiving a dividend of \$4, you sell the stock for \$800. What was your holding period return?

- **Q6**: What is the annual return on the DJ index given that

Stock	Price Jan 01 2013	Price Dec 31 2013	Event in the Year End
WFC	30	18	Split of 2 to 1
CVX	120	130	\$4 Dividend
CSCO	20	8	Split of 4 to 1

Questions

- **Q7**: Develop the mathematical relation between GA and HPR. Is there any mathematical relation between HPR and AA?

- **Q8**: In a two-day trade the returns are +100% and -50%. What are the AA and GA? Which one seems to be more accurate in reflecting the actual holding period return?

- **Q9**: The annual returns are R_1 = -10%, R_2 = -5%, and R_3 =7%. What is the holding period rate of return?

- **Q10:** Mr. A buys X dollar stocks on margin. Mrs. B sells short Y dollar stocks. Initial margin is 50%. Maintenance margin is 30%. Buying on margin: you get a margin call if the stock price drops by n%. Short selling: you get a margin call if the stock price appreciates by m%. Compare m and n and explain the results.

Questions

- **Q11**: You buy Citigroup on margin. The current price is $11. You have $110,000 and you borrow from your broker $110,000. The interest rate on the loan is 3% per year. Plot your annual profit (loss) as a function of the stock price prevailing one year from the purchase.

- **Q12**: You short sell Apple. That is, you sell 100 stocks you borrowed from the broker. The current share price is $450. You deposit the proceeds of the short sell as well as 50% of your equity in a margin account. Suppose that after two months you buy Apple back for $400 – what is the profit or loss as a % of the equity involved in this transaction?3

- **Q13:** What is short squeeze?

Market Efficiency vs. Market Anomalies

Case Studies: DFA and AQR

- This lecture covers topics related to the Dimensional Fund Advisors (DFA) as well as the AQR's momentum funds – both are HBS' case studies.
- Currently, DFA manages over and above 200 Billion Dollar – a fast growing investment corporation.
- Both DFA and AQR belong to the quantitative investment management category.
- That is, pick stocks based on financial ratios; not too much fundamental analysis.

Case Studies: Size, Value, Liquidity, and Momentum Effects

- The DFA's case study deals with the size and book to market (value) effects with some special treatment of market liquidity.
- The AQR's case deals with price momentum. The second pdf of this case study is basically a one line file indicating the momentum crash over the year 2009.

The Overall Agenda

- We start with a quick analysis of financial statements to recover useful financial ratios and quantities that define some of the strategies described throughout.
- We will present evidence from financial markets on the inability of active mutual funds to beat the market.
- We will then present performance of investment strategies termed market anomalies.

Financial Statements

- Every public firm releases three financial reports including the balance sheet, the income statement, and the case flows statement.
- There are three types of cash flows:
  a. CF from operating activities – a cash-based profit
  b. CF from investment activities
  c. CF from financing activities.
- The sum of these three components establishes the periodical change in the cash balance as appearing in the balance sheet.

Extracting Useful Quantities from Financial Statements

Let us visit yahoo/finance to look at Ford financial statements and estimate the following financial ratios and quantities:

- book-to-market
- earnings-to-price
- operating cash-flows-to-price
- dividend-to-price
- earnings multipliers – current, forward, and PEG

What is the Ultimate Question among Finance Practitioners?

- Could you beat the market?

  Beating the market means consistently accomplishing higher risk-adjusted return, relative to a simple investment in the market index.

- A valid question: good past performance – is it really skill or just a pure luck? Or, does past good performance predict good future performance (performance persistence)?

Market Efficiency

- Market efficiency summarizes the relation between stock prices and information available to investors.

- If markets are efficient, then all information is already in the stock price; the price is right. Or, the price at any particular moment reflects all available information.

- If the price is right it is impossible to "beat the market," except by luck.

- If the price isn't right, you can buy underpriced stocks and short sell overpriced stocks.

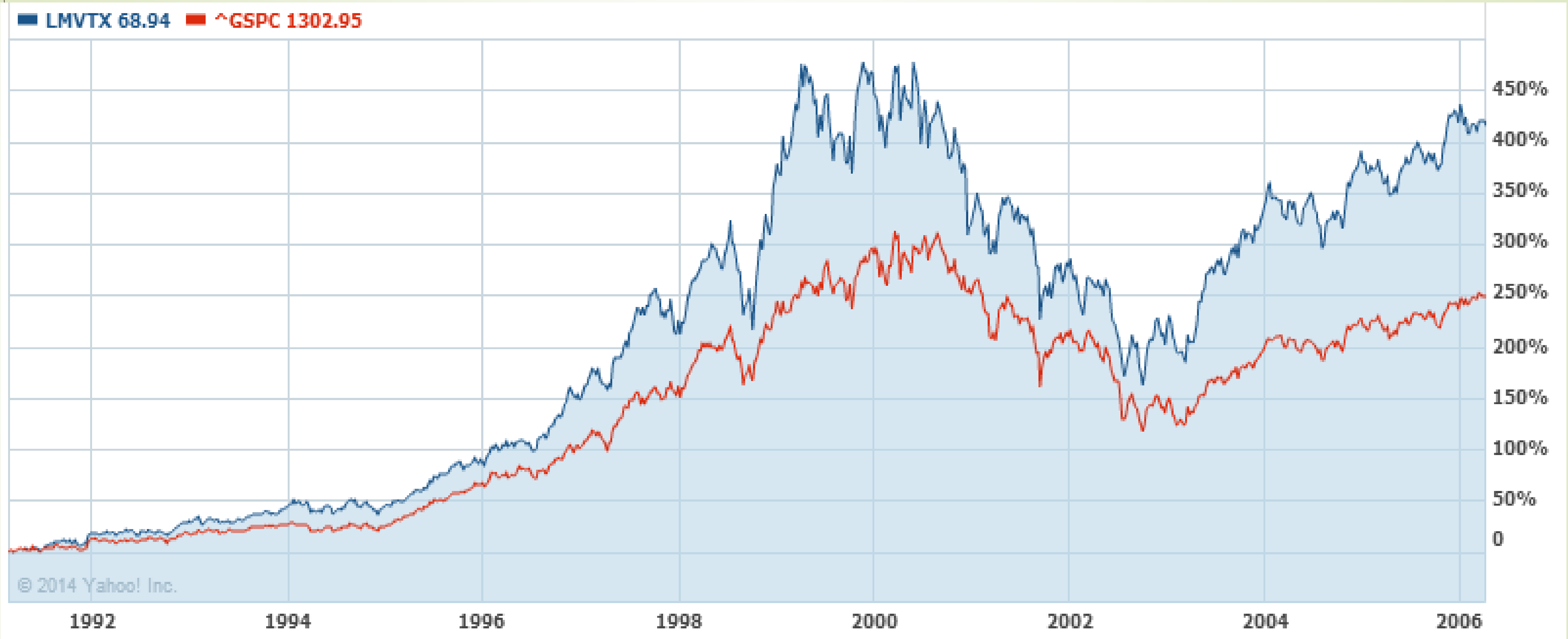
Are Markets Efficient?

- You will never know for sure.
- On one hand, it is difficult to believe that over the short run markets are able to digest all the huge amount of information affecting asset prices.
- On the other hand, sophisticated investment strategies have not been able to beat the market, for the most part.
- The empirical evidence is inconclusive.
- Fama and Shiller, two Noble Prize winners, share contradicting views.

Could Actively Managed Funds Beat the Market?

- Over the next slides we will show that actively managed funds DO NOT beat the market, on average, over the long run.
- Some do over short investment horizons.
- However, this could be due to luck, not skill, as performance virtually vanishes over longer periods – an inherent lack of persistence in performance.
- We start with Legg Mason Value to exhibit lack of persistence in performance.
- We will then display the lack of performance for the entire industry of equity mutual funds.

Bill Miller, Legg Mason fund: The best US trust fund 1991-2006

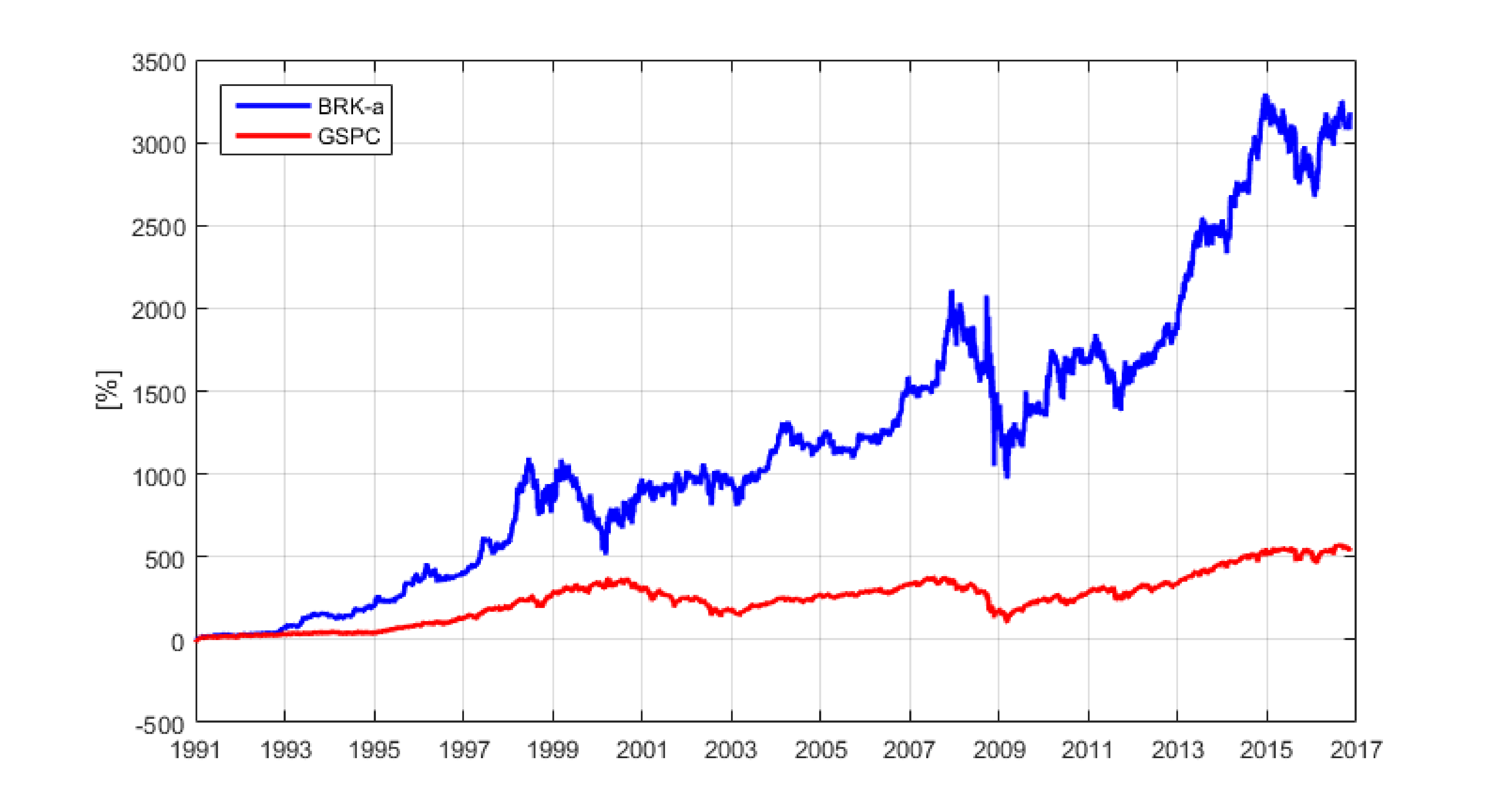


So would you Invest in Legg Mason in 2006?

- Probably you would!
- Based on empirical research - money is chasing performance.
- But will the investment be successful over the coming years?
- Coming next!

Bill Miller, Legg Mason fund:
The best US trust fund 2006-2016? Not any longer!

But Berkshire Hathaway has certainly Outperformed almost Consistently



Warren Buffett on Market Efficiency

"I think it is fascinating how the ruling orthodoxy can cause a lot of people to think the earth is flat. Investing in a market where people believe in efficiency is like playing bridge with someone who's been told it doesn't do any good to look at the cards."

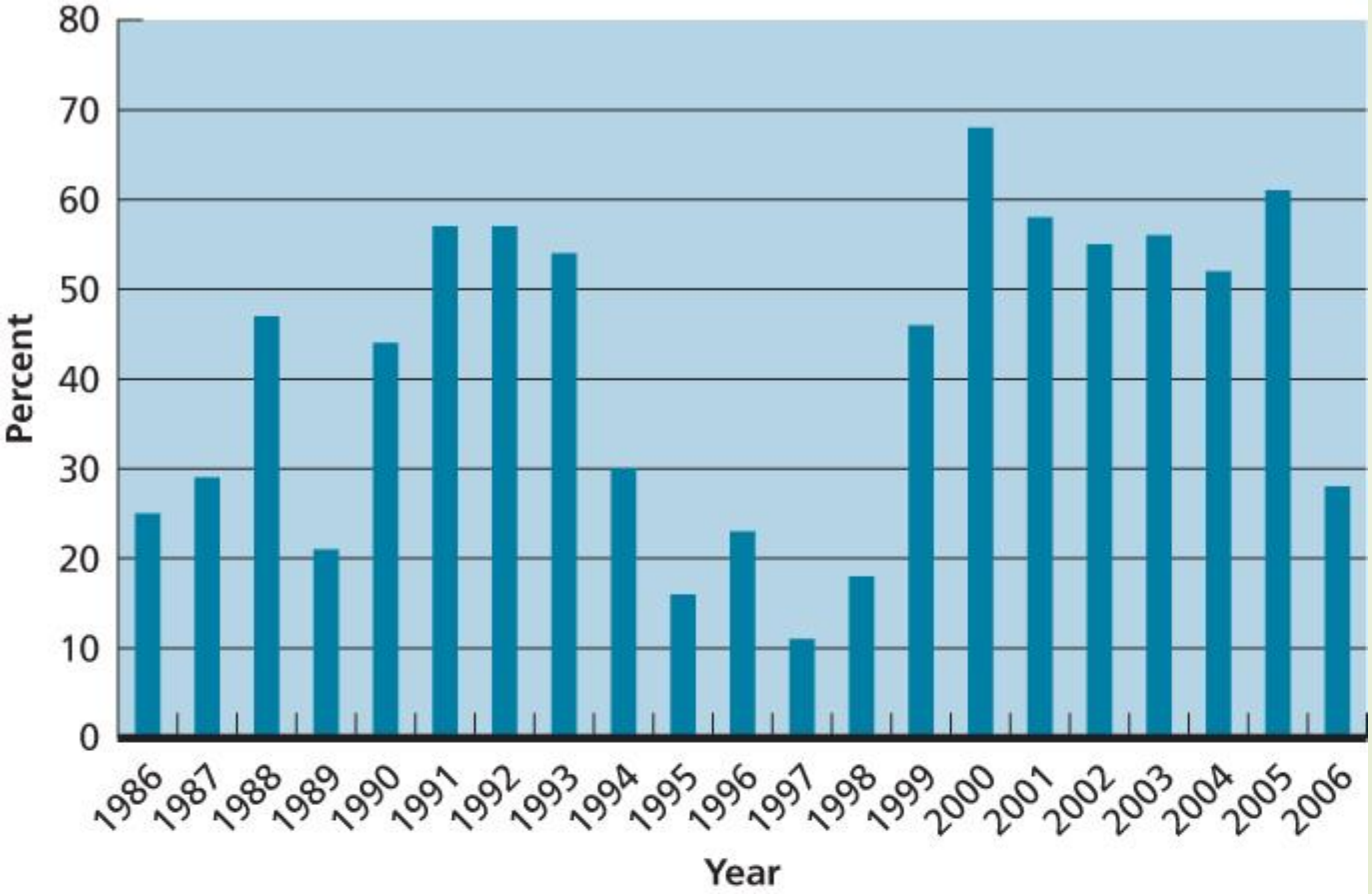
Understanding Buffett's Outstanding Performance

Frazzini, Kabiller, and Pedersen, 2013:

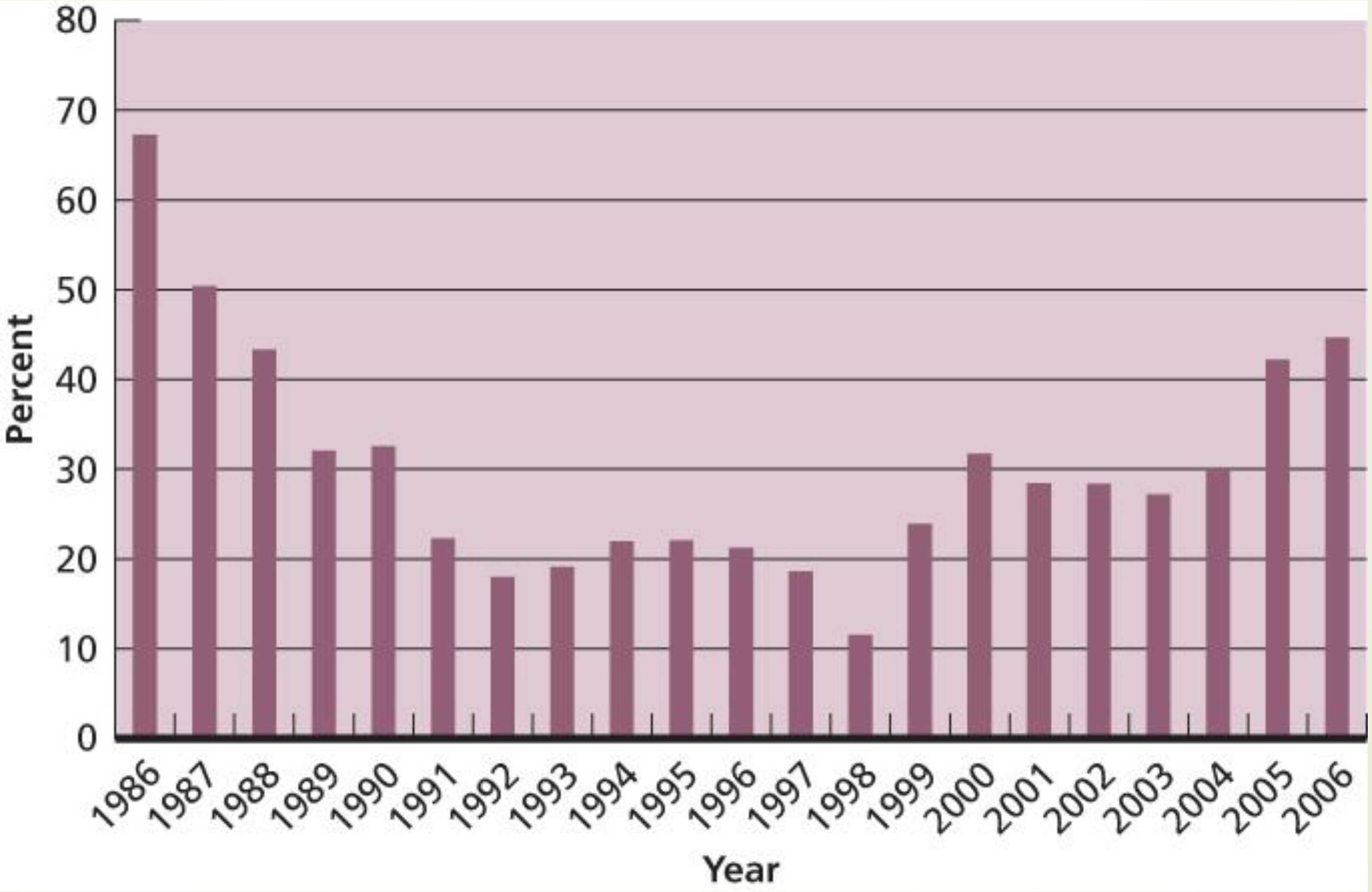
- Buffett's returns appear to be neither luck nor magic, but, rather, reward for the use of leverage combined with a focus on **cheap**, **safe**, **quality** stocks.

- Decomposing Berkshires' portfolio into ownership in publicly traded stocks versus wholly-owned private companies, the former performs the best, suggesting that Buffett's returns are more due to stock selection than to his effect on management.

And what about performance in the overall universe of U.S. equity mutual funds?



Here is a Longer Horizon (10 years) Perspective



Summary: The Lack of Performance over Various Investment Horizons

Length of each Investment Period (Years)	Span	Number of Investment Periods	Number of Investment Periods	Percent (%)	Number of Investment Periods	Percent (%)
1	1977-2006	30	14	46.7	2	6.7
3	1979-2006	28	13	46.4	2	7.1
5	1981-2006	26	9	34.6	2	7.7
10	1986-2006	21	2	9.5	0	0.0

Active versus Passive Investment Management

- It is fairly safe to conclude that active management does not beat the market, on average, on a consistent basis.
- This applies to financial markets around the world - no exceptions.

Mitigating Circumstances

- Mutual funds have to reserve some amount of liquidity to accommodate potential redemptions, which could hurt performance.
- The overall picture looks better based on gross returns (adding back management fees and trading costs).
- The figures aggregate all investable funds, while one could perhaps detect profitable strategies among sub groups of mutual funds such as "hot hands" and "smart money."
- One could also exploit market wide economic variables (such as the dividend yield, the default spread, the yield spread, etc.) to identify outperforming funds.

Hedge Funds

- While it is a common knowledge that mutual funds do not beat the market on average – there is some solid evidence that hedge funds do. But information on hedge funds is incomplete.
- HFs are private investment partnerships of high wealth individuals or qualified investors (pension funds).
- In the US, HFs are usually up to 500-investor partnership.
- General partners typically have significant fraction of their own wealth invested in the fund.
- A comparison between hedge funds and mutual funds is useful for further understanding what hedge funds are.

Mutual vs. Hedge Funds

- Regulation: MFs are regulated by the SEC. Their reports are detailed and strict. On the other hand, U.S. hedge funds are exempt from many of the standard registration and reporting requirements because they only accept accredited investors.
- In 2010, regulations were enacted in the US and European Union, which introduced additional hedge fund reporting requirements. These included the U.S.'s Dodd-Frank Wall Street Reform Act and European Alternative Investment Fund Managers Directive.
- In general, hedge funds are more prone to fraud. Fortunately, there have not been many such incidences. The most notable one is Bernard Madoff who committed a Ponzi scheme. Total loss about $ 17 billion – the largest financial fraud in US history.
- A Ponzi scheme is a fraudulent investment operation where the operator pays returns to its investors from new capital paid to the operators by new investors, rather than from profit earned by the operator.

Mutual vs. Hedge Funds

- Minimum investment: MF small; HF large (around $1 M)
- Investors: MFs unlimited; HFs limited partnership.
- Availability: MFs are publicly available; HFs to accredited investors.
- Liquidity: MFs daily liquidity and redemptions; HFs liquidity varies from monthly to annually.

Mutual vs. Hedge Funds

- Short selling: MFs up to 30% of profit from shot sale; HFs – unlimited.
- Fees: MFs limits imposed by the SEC; HFs – no limits. Plus, HFs could charge asymmetric management fees based on performance. MFs cannot.
- One major difference is the nature of trading strategies. While mutual funds typically undertake solid long positions, hedge funds are involved in pair trading, buying and writing derivatives, day trading, etc. The nature of trading coming up from hedge funds is more speculative and sophisticated.
- Correspondingly, mutual fund returns are typically highly correlated with the market returns, while hedge funds could record low correlation (the ambition is to be market neutral).

Market Anomalies

- We next move to analyze several prominent market anomalies.
- Anomaly is defined to be a deviation from a rule or a trend.
- In finance, the rule is the CAPM.
- Return patterns that are unexplained by the CAPM are considered to be anomalous.
- We study the size, value, price momentum, earnings momentum, and volatility effects.
- There are other anomalies.

Anomaly I: The size effect

- *Size effect*: higher average returns on small stocks than large stocks. Beta cannot explain the difference.

Table I

Average Returns, Post-Ranking βs and Average Size For Portfolios Formed on Size and then β: Stocks Sorted on ME (Down) then Pre-Ranking β (Across): July 1963 to December 1990

Portfolios are formed yearly. The breakpoints for the size (ME, price times shares outstanding) deciles are determined in June of year t (t = 1963–1990) using all NYSE stocks on CRSP. All NYSE, AMEX, and NASDAQ stocks that meet the CRSP-COMPUSTAT data requirements are allocated to the 10 size portfolios using the NYSE breakpoints. Each size decile is subdivided into 10 β portfolios using pre-ranking βs of individual stocks, estimated with 2 to 5 years of monthly returns (as available) ending in June of year t. We use only NYSE stocks that meet the CRSP-COMPUSTAT data requirements to establish the β breakpoints. The equal-weighted monthly returns on the resulting 100 portfolios are then calculated for July of year t to June of year $t + 1$.

The post-ranking βs use the full (July 1963 to December 1990) sample of post-ranking returns for each portfolio. The pre- and post-ranking βs (here and in all other tables) are the sum of the slopes from a regression of monthly returns on the current and prior month's returns on the value-weighted portfolio of NYSE, AMEX, and (after 1972) NASDAQ stocks. The average return is the time-series average of the monthly equal-weighted portfolio returns, in percent. The average size of a portfolio is the time-series average of monthly averages of ln(ME) for stocks in the portfolio at the end of June of each year, with ME denominated in millions of dollars.

The average number of stocks per month for the size-β portfolios in the smallest size decile varies from 70 to 177. The average number of stocks for the size-β portfolios in size deciles 2 and 3 is between 15 and 41, and the average number for the largest 7 size deciles is between 11 and 22.

The All column shows statistics for equal-weighted size-decile (ME) portfolios. The All row shows statistics for equal-weighted portfolios of the stocks in each β group.

	All	Low-β	β-2	β-3	β-4	β-5	β-6	β-7	β-8	β-9	High-β
Panel A: Average Monthly Returns (in Percent)											
All	1.25	1.34	1.29	1.36	1.31	1.33	1.28	1.24	1.21	1.25	1.14
Small-ME	1.52	1.71	1.57	1.79	1.61	1.50	1.50	1.37	1.63	1.50	1.42
ME-2	1.29	1.25	1.42	1.36	1.39	1.65	1.61	1.37	1.31	1.34	1.11
ME-3	1.24	1.12	1.31	1.17	1.70	1.29	1.10	1.31	1.36	1.26	0.76
ME-4	1.25	1.27	1.13	1.54	1.06	1.34	1.06	1.41	1.17	1.35	0.98
ME-5	1.29	1.34	1.42	1.39	1.48	1.42	1.18	1.13	1.27	1.18	1.08
ME-6	1.17	1.08	1.53	1.27	1.15	1.20	1.21	1.18	1.04	1.07	1.02
ME-7	1.07	0.95	1.21	1.26	1.09	1.18	1.11	1.24	0.62	1.32	0.76
ME-8	1.10	1.09	1.05	1.37	1.20	1.27	0.98	1.18	1.02	1.01	0.94
ME-9	0.95	0.98	0.88	1.02	1.14	1.07	1.23	0.94	0.82	0.88	0.59
Large-ME	0.89	1.01	0.93	1.10	0.94	0.93	0.89	1.03	0.71	0.74	0.56

Anomaly II: The value effect

- *Value effect*: higher average returns on value stocks than growth stocks. Beta cannot explain the difference.
- *Value firms*: Firms with high E/P, B/P, D/P, or CF/P. The notion of value is that physical assets can be purchased at low prices.
- *Growth firms*: Firms with low ratios. The notion is that high price relative to fundamentals reflects capitalized growth opportunities.

Table I

Summary Statistics and Three-Factor Regressions for Simple Monthly Percent Excess Returns on 25 Portfolios Formed on Size and BE/ME: 7/63–12/93, 366 Months

R_f is the one-month Treasury bill rate observed at the beginning of the month (from CRSP). The explanatory returns R_M, SMB, and HML are formed as follows. At the end of June of each year t (1963–1993), NYSE, AMEX, and Nasdaq stocks are allocated to two groups (small or big, S or B) based on whether their June market equity (ME, stock price times shares outstanding) is below or above the median ME for NYSE stocks. NYSE, AMEX, and Nasdaq stocks are allocated in an independent sort to three book-to-market equity (BE/ME) groups (low, medium, or high; L, M, or H) based on the breakpoints for the bottom 30 percent, middle 40 percent, and top 30 percent of the values of BE/ME for NYSE stocks. Six size-BE/ME portfolios (S/L, S/M, S/H, B/L, B/M, B/H) are defined as the intersections of the two ME and the three BE/ME groups. Value-weight monthly returns on the portfolios are calculated from July to the following June. SMB is the difference, each month, between the average of the returns on the three small-stock portfolios (S/L, S/M, and S/H) and the average of the returns on the three big-stock portfolios (B/L, B/M, and B/H). HML is the difference between the average of the returns on the two high-BE/ME portfolios (S/H and B/H) and the average of the returns on the two low-BE/ME portfolios (S/L and B/L). The 25 size-BE/ME portfolios are formed like the six size-BE/ME portfolios used to construct SMB and HML, except that quintile breakpoints for ME and BE/ME for NYSE stocks are used to allocate NYSE, AMEX, and Nasdaq stocks to the portfolios.

BE is the COMPUSTAT book value of stockholders' equity, plus balance sheet deferred taxes and investment tax credit (if available), minus the book value of preferred stock. Depending on availability, we use redemption, liquidation, or par value (in that order) to estimate the book value of preferred stock. The BE/ME ratio used to form portfolios in June of year t is then book common equity for the fiscal year ending in calendar year $t-1$, divided by market equity at the end of December of $t-1$. We do not use negative BE firms, which are rare prior to 1980, when calculating the breakpoints for BE/ME or when forming the size-BE/ME portfolios. Also, only firms with ordinary common equity (as classified by CRSP) are included in the tests. This means that ADR's, REIT's, and units of beneficial interest are excluded.
The market return R_M is the value-weight return on all stocks in the size-BE/ME portfolios, plus the negative BE stocks excluded from the portfolios.

	Book-to-Market Equity (BE/ME) Quintiles									
Size	Low	2	3	4	High	Low	2	3	4	High
	Panel A: Summary Statistics									
	Means					Standard Deviations				
Small	0.31	0.70	0.82	0.95	1.08	7.67	6.74	6.14	5.85	6.14
2	0.48	0.71	0.91	0.93	1.09	7.13	6.25	5.71	5.23	5.94
3	0.44	0.68	0.75	0.86	1.05	6.52	5.53	5.11	4.79	5.48
4	0.51	0.39	0.64	0.80	1.04	5.86	5.28	4.97	4.81	5.67
Big	0.37	0.39	0.36	0.58	0.71	4.84	4.61	4.28	4.18	4.89

The Value Effect: The International Evidence

Table III

Annual Dollar Returns in Excess of U.S. T-Bill Rate for Market, Value, and Growth Portfolios: 1975–1995

Value and growth portfolios are formed on book-to-market equity (B/M), earnings/price (E/P), cashflow/price (C/P), and dividend/price (D/P), as described in Table II. We denote value (high) and growth (low) portfolios by a leading H or L; the difference between them is H − L. The first row for each country is the average annual return. The second is the standard deviation of the annual returns (in parentheses) or the t-statistic testing whether H − L is different from zero [in brackets].

	Market	HB/M	LB/M	H-LB/M	HE/P	LE/P	H-LE/P	HC/P	LC/P	H-LC/P	HD/P	LD/P	H-LD/P
U.S.	9.57	14.55	7.75	6.79	14.09	7.38	6.71	13.74	7.08	6.66	11.75	8.01	3.73
	(14.64)	(16.92)	(15.79)	[2.17]	(18.10)	(15.23)	[2.28]	(16.73)	(15.99)	[2.08]	(13.89)	(17.04)	[1.22]
Japan	11.88	16.91	7.06	9.85	14.14	6.67	7.47	14.95	5.66	9.29	16.81	7.27	9.54
	(28.67)	(27.74)	(30.49)	[3.49]	(26.10)	(27.62)	[4.00]	(31.59)	(29.22)	[3.03]	(35.01)	(27.51)	[2.53]
U.K.	15.33	17.87	13.25	4.62	17.46	14.81	2.65	16.41	14.51	3.89	15.89	12.99	2.90
	(28.62)	(30.03)	(27.94)	[1.08]	(32.32)	(27.00)	[0.83]	(35.11)	(26.55)	[0.85]	(32.18)	(26.32)	[0.72]
France	11.26	17.10	9.46	7.64	15.68	8.70	6.98	16.17	9.30	6.86	15.12	6.25	8.88
	(32.35)	(36.60)	(30.88)	[2.08]	(37.05)	(32.35)	[2.16]	(36.92)	(31.26)	[2.29]	(30.06)	(33.16)	[2.48]
Germany	9.88	12.77	10.01	2.75	11.13	10.58	0.55	13.28	5.14	8.13	9.99	10.42	−0.43
	(31.36)	(30.35)	(32.75)	[0.92]	(24.62)	(34.82)	[0.14]	(29.05)	(26.94)	[2.62]	(24.88)	(34.42)	[−0.10]
Italy	8.11	5.45	11.44	−5.99	7.62	12.99	−5.37	11.05	0.37	10.69	10.07	12.68	−2.61
	(43.77)	(35.53)	(50.65)	[−0.91]	(42.36)	(54.68)	[−0.84]	(43.52)	(38.42)	[1.73]	(38.28)	(56.66)	[−0.33]
Netherlands	13.30	15.77	13.47	2.30	14.37	9.26	5.11	11.66	11.84	−0.19	13.47	13.05	0.41
	(18.81)	(33.07)	(21.01)	[0.44]	(21.07)	(20.48)	[1.04]	(33.02)	(23.26)	[−0.03]	(21.38)	(30.81)	[0.07]
Belgium	12.62	14.90	10.51	4.39	15.12	12.90	2.22	16.46	12.03	4.44	15.16	12.26	2.91
	(25.88)	(28.62)	(27.63)	[1.99]	(30.47)	(27.88)	[0.78]	(28.84)	(25.57)	[1.27]	(26.47)	(29.26)	[1.29]
Switzerland	11.07	13.84	10.34	3.49	12.59	11.04	1.54	12.32	9.78	2.53	12.62	10.44	2.18
	(27.21)	(30.00)	(28.57)	[0.80]	(31.44)	(28.81)	[0.36]	(36.58)	(27.82)	[0.41]	(31.00)	(27.83)	[0.63]
Sweden	12.44	20.61	12.59	8.02	20.61	12.42	8.19	17.08	12.50	4.58	16.15	11.32	4.83
	(24.91)	(38.31)	(26.26)	[1.16]	(42.43)	(24.76)	[1.03]	(30.56)	(23.58)	[0.90]	(29.55)	(25.13)	[1.05]
Australia	8.92	17.62	5.30	12.32	15.64	5.97	9.67	18.32	4.03	14.29	14.62	6.83	7.79
	(26.31)	(31.03)	(27.32)	[2.41]	(28.19)	(28.89)	[1.71]	(29.08)	(27.46)	[2.85]	(28.43)	(28.57)	[1.65]
Hong Kong	22.52	26.51	19.35	7.16	27.04	22.05	4.99	29.33	20.24	9.09	23.66	23.30	0.35
	(41.96)	(48.68)	(40.21)	[1.35]	(44.83)	(40.81)	[0.82]	(46.24)	(42.72)	[1.37]	(38.76)	(42.05)	[0.09]
Singapore	13.31	21.63	11.96	9.67	15.21	13.12	2.09	13.42	8.03	5.39	10.64	13.10	−2.46
	(27.29)	(36.89)	(27.71)	[2.36]	(29.55)	(34.68)	[0.65]	(26.24)	(28.92)	[1.49]	(22.01)	(33.93)	[−0.45]

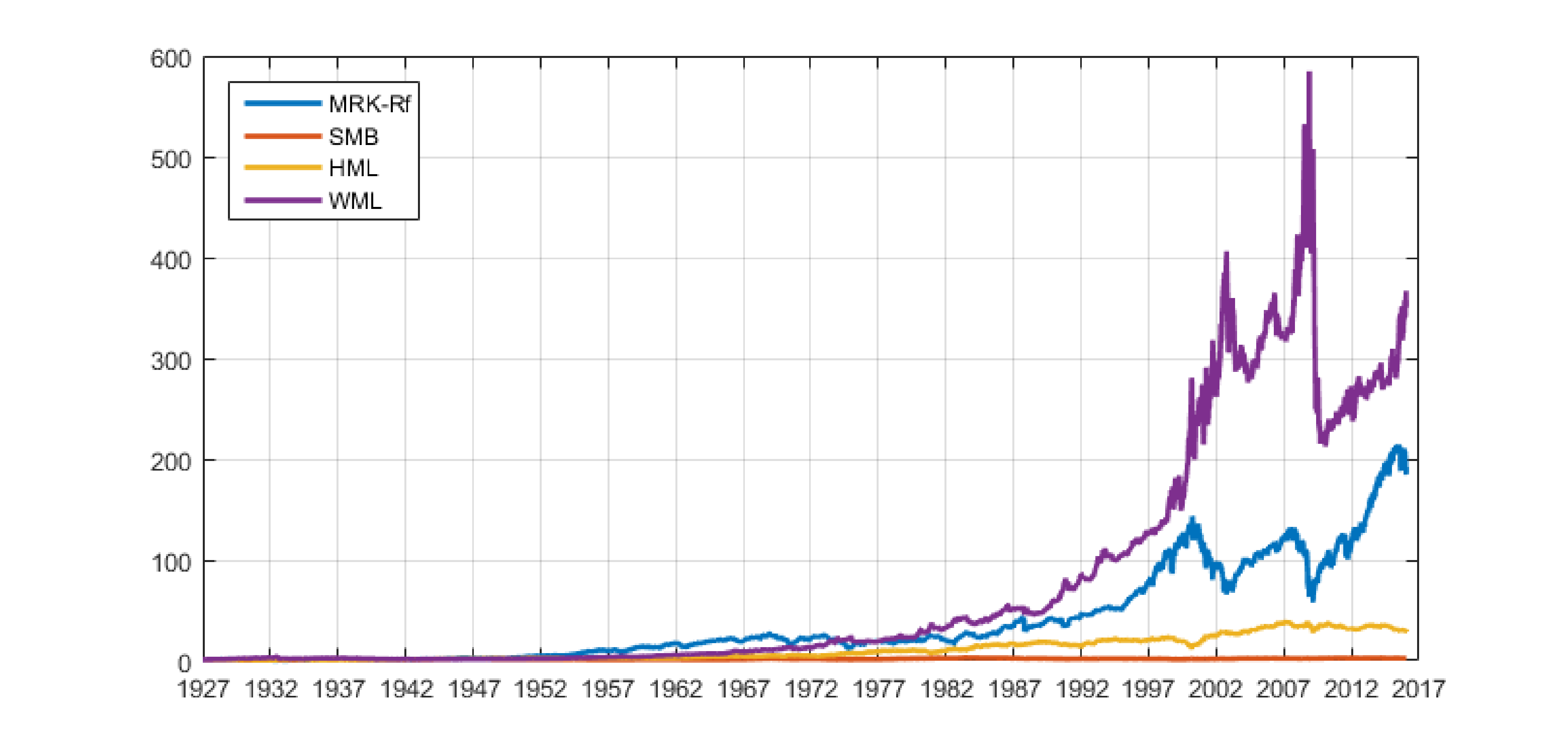
Anomaly III: Price Momentum

- Jegadeesh and Titman (1993) found that recent past winners (stocks with high returns in the last 3, 6, 9, and 12 months) outperform recent past losers.

- Keep in mind that practitioners already were using momentum strategies prior to the academic discovery.

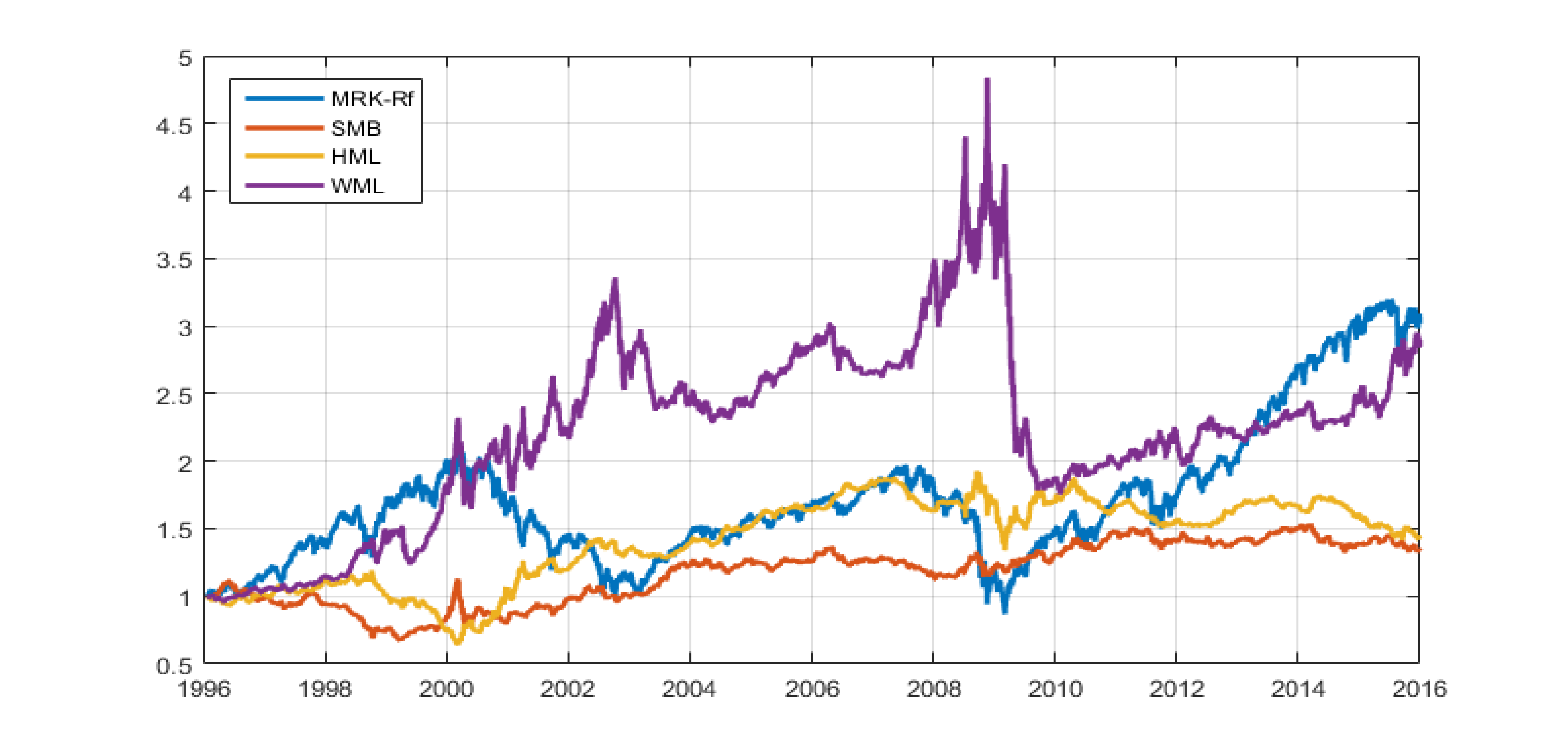
- Take a look at the figures.

Cumulative Return of 1 USD Invested 1927 Jan-2016 Jan



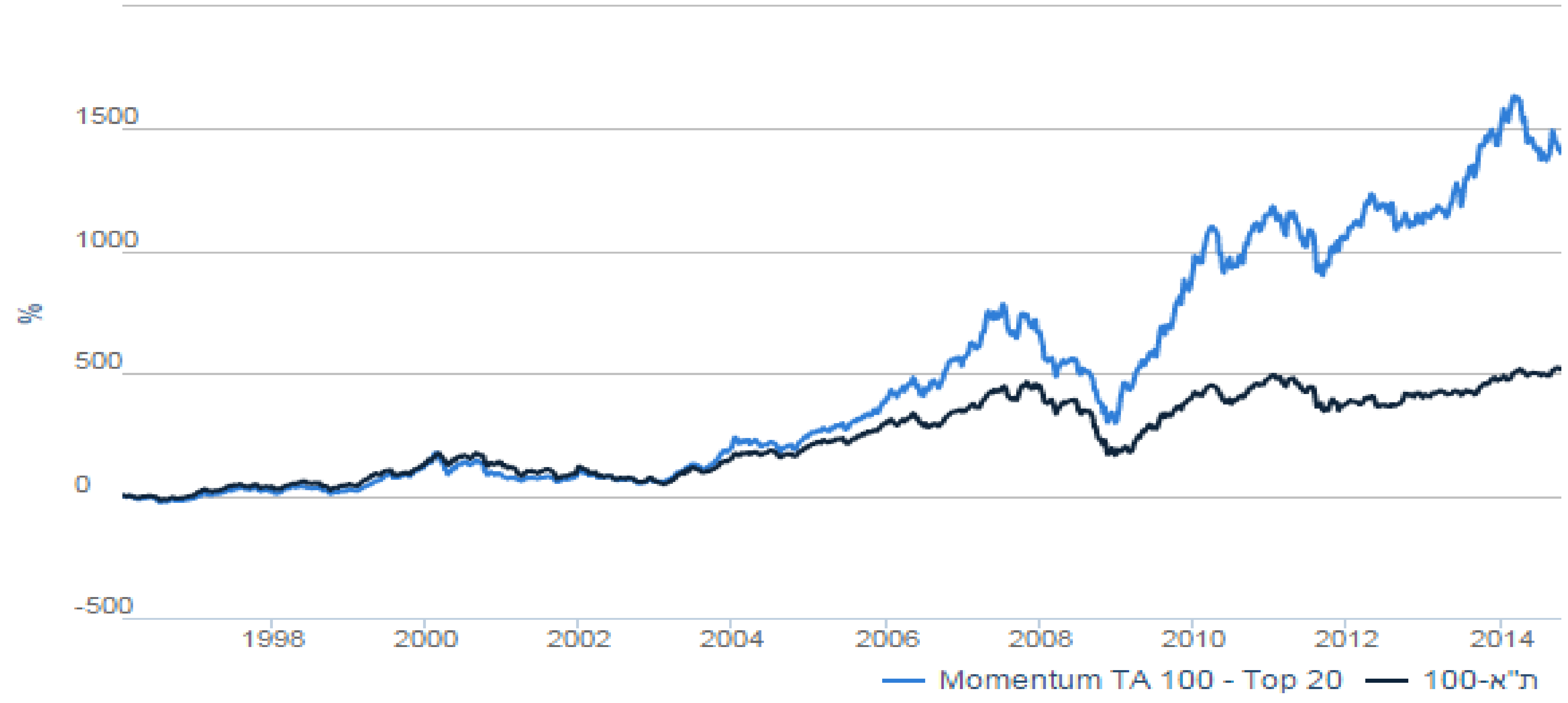
Mkt-RF	SMB	HML	WML
$ 192.27	$2.4	$ 30.38	$ 350.16

Cumulative Return of 1 USD Invested Last 2 Decades (1996 Jan– 2016 Jan)



Mkt-RF	SMB	HML	WML
$ 2.86	$ 1.49	$ 1.51	$ 2.06

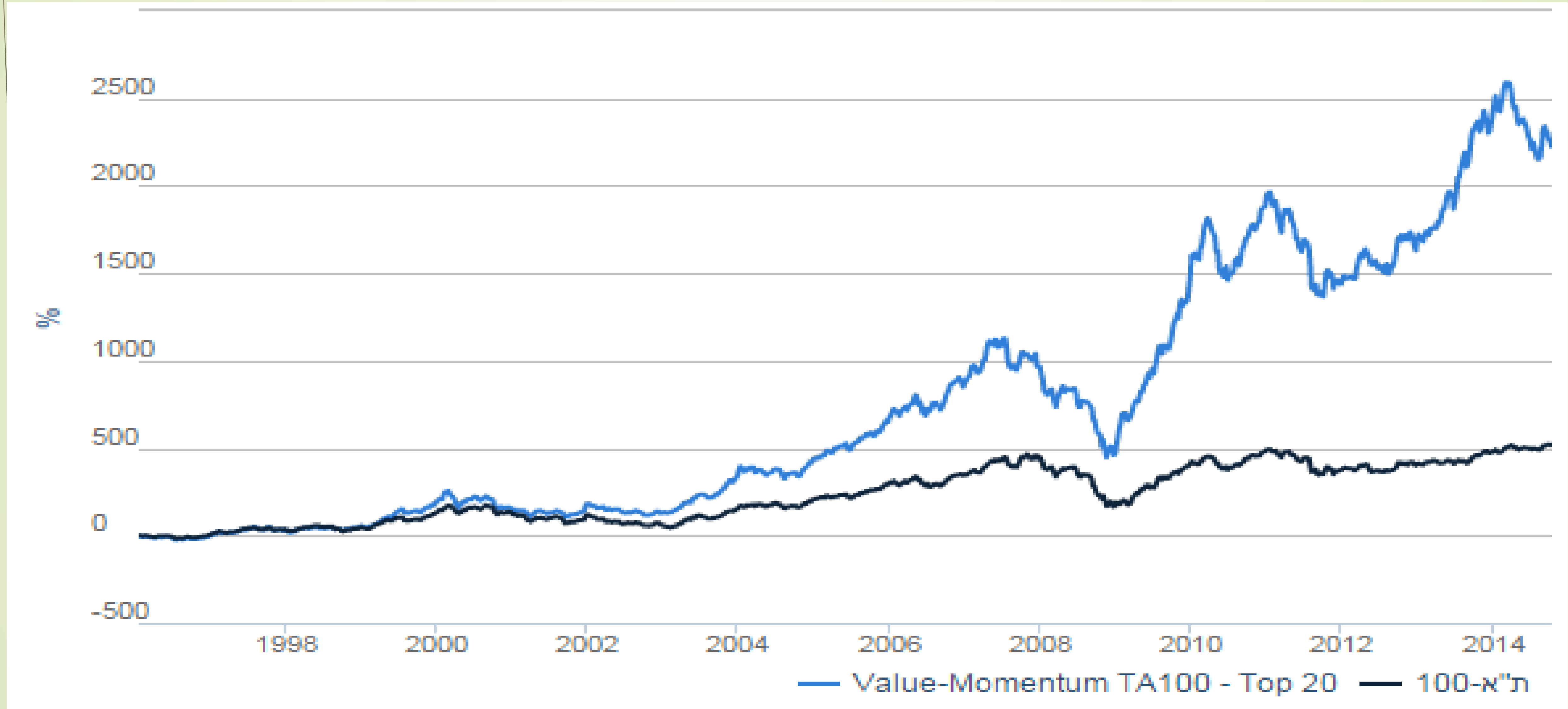
In Israel: Momentum TA100 – Top 20



Momentum TA100 – Top 20

As of 13/10/2014	Year			5 Years				10 Years				Full Period			
Asset Name	Annual Return (%)	Annual Std (%)	Sharpe Ratio	Annual Return (%)	Annual Std (%)	Draw Down 12 (%)	Sharpe Ratio	Annual Return (%)	Annual Std (%)	Draw Down 12 (%)	Sharpe Ratio	Annual Return (%)	Annual Std (%)	Draw Down 12 (%)	Sharpe Ratio
Momentum TA 100 - Top 20	-1.8	13.3	-0.21	11.1	18.1	-14.4	0.49	17.6	22.6	-53.6	0.63	16.2	23.1	-53.6	0.41
ת"א-100	11.2	8.9	1.14	6.1	15.3	-20.6	0.25	8.9	19.2	-54.6	0.29	10.6	20.4	-54.6	0.19
Excess Return	-13			5				8.8				5.6			

Value - Momentum TA100 – Top 20



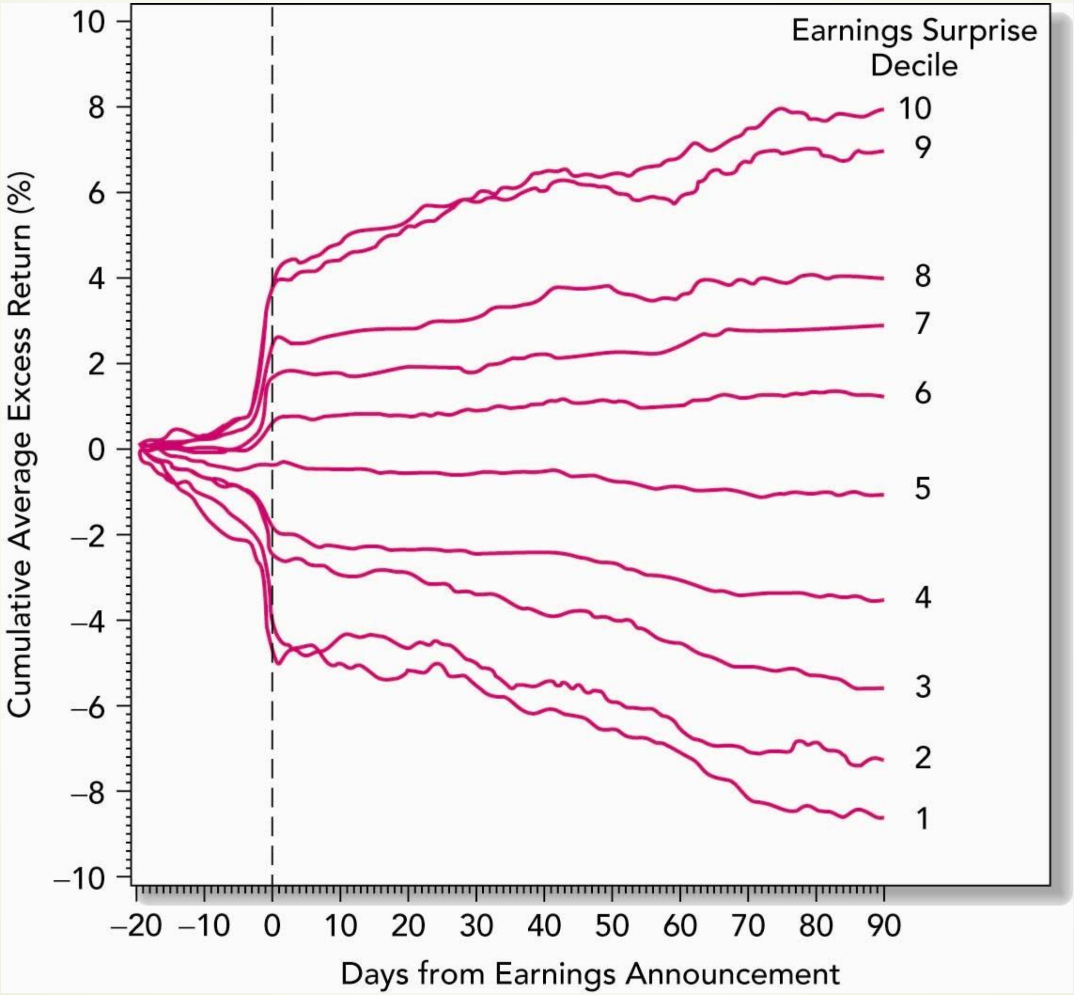
Value - Momentum TA100 – Top 20

As of 13/10/2014	Year			5 Years				10 Years				Full Period			
Asset Name	Annual Return (%)	Annual Std (%)	Sharpe Ratio	Annual Return (%)	Annual Std (%)	Draw Down 12 (%)	Sharpe Ratio	Annual Return (%)	Annual Std (%)	Draw Down 12 (%)	Sharpe Ratio	Annual Return (%)	Annual Std (%)	Draw Down 12 (%)	Sharpe Ratio
Value-Momentum TA100 - Top 20	-4.2	12.7	-0.41	12.2	17.5	-24.8	0.57	18	21.8	-54.5	0.68	19.1	22.4	-54.5	0.55
ת"א-100	11.2	8.9	1.14	6.1	15.3	-20.6	0.25	8.9	19.2	-54.6	0.29	10.6	20.4	-54.6	0.19
Excess Return	-15.4			6.1				9.2				8.5			

Momentum Robustness

- Momentum is fairly robust in a cross-industry analysis, cross-country analysis, cross-style analysis, and within other countries virtually all over the world excluding Japan.
- Momentum also seems to appear in bonds, currencies, and commodities, as well as in mutual funds and hedge funds.
- The momentum phenomena has invoked many academic studies trying to understand its sources and payoff structure.
- Momentum could be rationally based, it could be triggered by behavioral biases, or it could be attributable to under-reaction to financial news.
- But in 2009 – momentum delivers a -85% payoff!!!
- There are some other episodes of momentum crash.

Anomaly IV: Earnings Momentum: Cumulative Returns in Response to Earnings Announcements



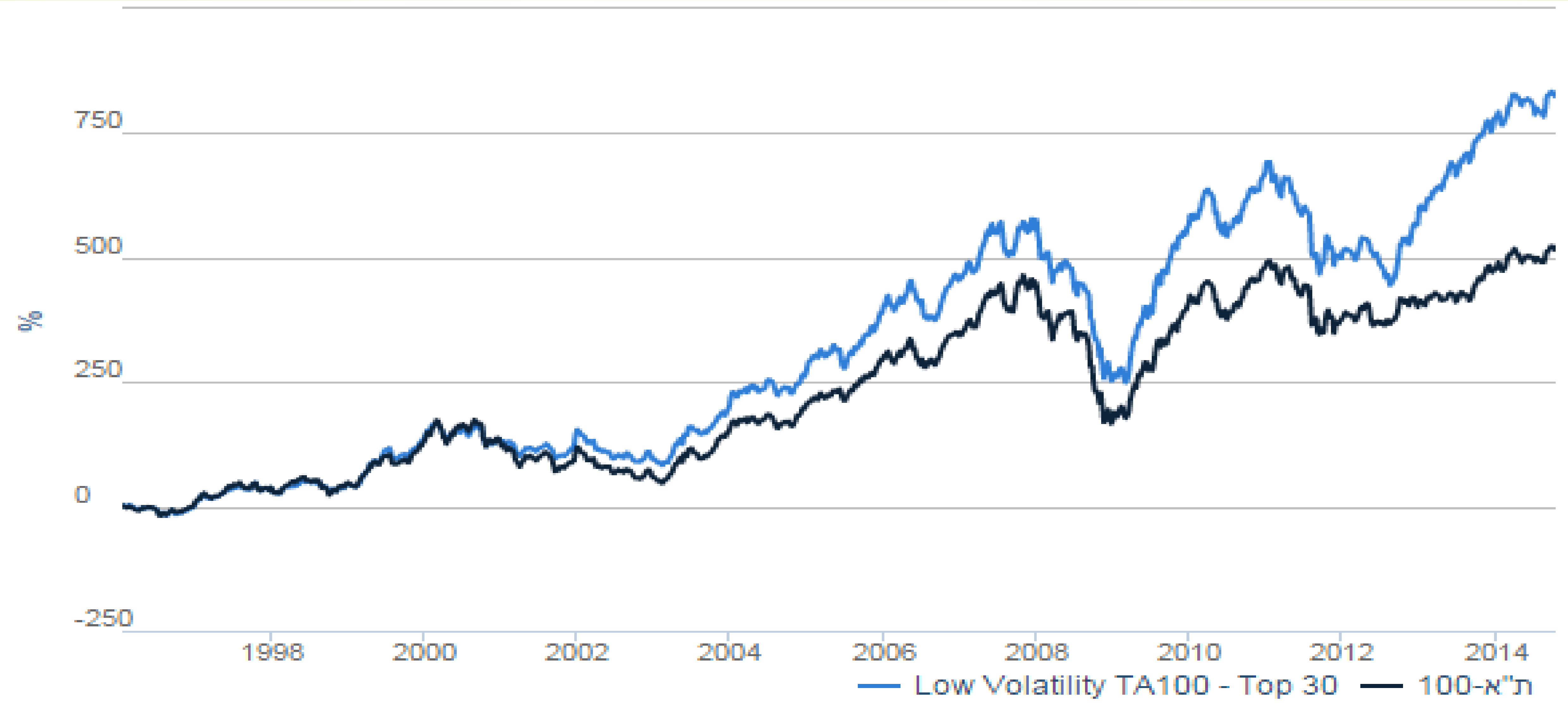
Anomaly VI: Minimum Volatility

- The idea here is to twofold:

  a. invest in stocks with the lowest volatility.

  b. invest in the global minimum volatility portfolio.

Volatility - TA100 – Top 30



ETFs Revisited: Investing in Anomalies

- USMV: An ETF trading minimum volatility stocks.
- PKW: An ETF buying at least 90% of stocks of companies that repurchase their own stocks.
- AMOMX: An ETF investing in large and mid-cap momentum stocks.
- IWN: An ETF investing in small cap value stocks.
- CSD: Investing in spin-offs (e.g., PayPal).

Economic Links

- Often times investors do not recognize economic links between economically related firms.

- Something that can be exploited to establish profitable strategies due to such limited attention.

- Here is a striking example.

Economic Links

- Coastcast Corporation was a leading manufacturer of golf club heads.
- Since 1993 its major customer (50% of the sales) had been Callaway Golf Corporation
- On June 2001, Callaway was downgraded – expected EPS was down from 70 cents per share to only 35.
- On the very same day the Callaway's stock price is down about 30% which makes sense.
- However, no impact on Coastcast's stock price.
- The ultimate adjustment occurred only two months later.

Appendix I: Investment Analysis

- There are three major types of investment analysis:
  a. Fundamental analysis
  b. Quantitative analysis
  c. Technical analysis
- The study of market anomalies involves both fundamental and technical indicators.

Fundamental analysis: Overview

- For trading a stock, it involves the analysis of the firm's revenues, profitability, cash flows, quality of management, competitive advantages, the industry affiliation, and the market as a whole.
- Several relevant questions:
  - **Growth**: Are profits, margins, and free cash flow all increasing?
  - **Valuation**: Is share price growing in line with earnings per share?
  - **Opportunities**: Is return on equity increasing while debt to equity declines?
  - **Dividends**: Are dividends consistently growing in a sustainable way?
- When applied to currencies and government issued securities, it focuses on the overall state of the economy, interest rates, production growth, aggregate earnings, and the quality of primary decision makers.

Fundamental Analysis: Is the Price Right?

- Fundamental analysis maintains that markets may misprice a security in the short run but that the "correct" price will eventually be reached.
- Profits thus can be made by purchasing under-priced (market price below the right price) securities and selling over-priced (market price above the right price) securities - then waiting for the market to recognize its "mistake" and converge to the right price.
- For instance, low PEG ratio stocks could be thought of as under-priced, where PEG is the price to earnings ratio divided by expected growth rate.

Quantitative Finance

- Started in the U.S. in the 1970s as some astute investors began using mathematical formulas to price stocks, bonds, and options.
- Two notable examples:
  - Harry Markowitz's 1952 was one of the first papers to formally adapt mathematical concepts to finance.
  - Markowitz formalized a notion of mean variance efficiency which is covered in the next chapter.
- Fischer Black and Myron Scholes developed the Black–Scholes model, which provides a solution for the fair price for a European call option.

Technical Analysis

- A security analysis methodology for forecasting the direction of prices through the study of past market data, primarily price and volume.
- While fundamental analysts examine earnings, dividends, new products, research and the like, technical analysts seek to identify price patterns and market trends in financial markets and attempt to exploit those patterns.
- The basic idea: **the trend is your friend**.
- Technicians using charts search for archetypal price chart patterns, such as the well-known head and shoulders or double top/bottom reversal patterns, study technical indicators, moving averages, and look for forms such as lines of support, resistance, channels, and more obscure formations such as flags, pennants, balance days and cup and handle patterns.

Technical Analysis

- Unlike fundamental analysis, technical analysis holds that prices already reflect all trends before investors are aware of them. Uncovering those trends is what technical indicators are designed to do.
- Most large brokerage, trading group, or financial institutions will typically have both a technical analysis and fundamental analysis team.

Appendix II: Three Forms of Market Efficiency

- **Weak-form efficient market**
  A market in which past prices and volume figures are of no use in establishing outperforming trading strategies.

- **Semi-strong-form efficient market**
  A market in which all publicly available information is of no use in beating the market.

- **Strong-form efficient market**
  A market in which information of any kind, public or private, is of no use in beating the market.

Strong-form information set:
All information of any kind, public or private.

Semi-strong-form information set:
All publicly available information.

Weak-form information set:
Past price and volume.

End-of-Chapter Questions

- **Q1**: What is pair trading?

- **Q2**: Assume that a strategy that buys low volatility stocks and sells high volatility stocks consistently outperforms the market. Does this evidence establish violation of market efficiency? If yes, what sorts of market efficiency are being violated? Does this evidence violate the risk-return tradeoff?

- **Q3**: What is “short term reversal”? Does the concept violates market efficiency? If yes, what sorts of market efficiency are being violated?

- **Q4**: explain the relationship between the book-to-market ratio, value stocks, and growth stocks

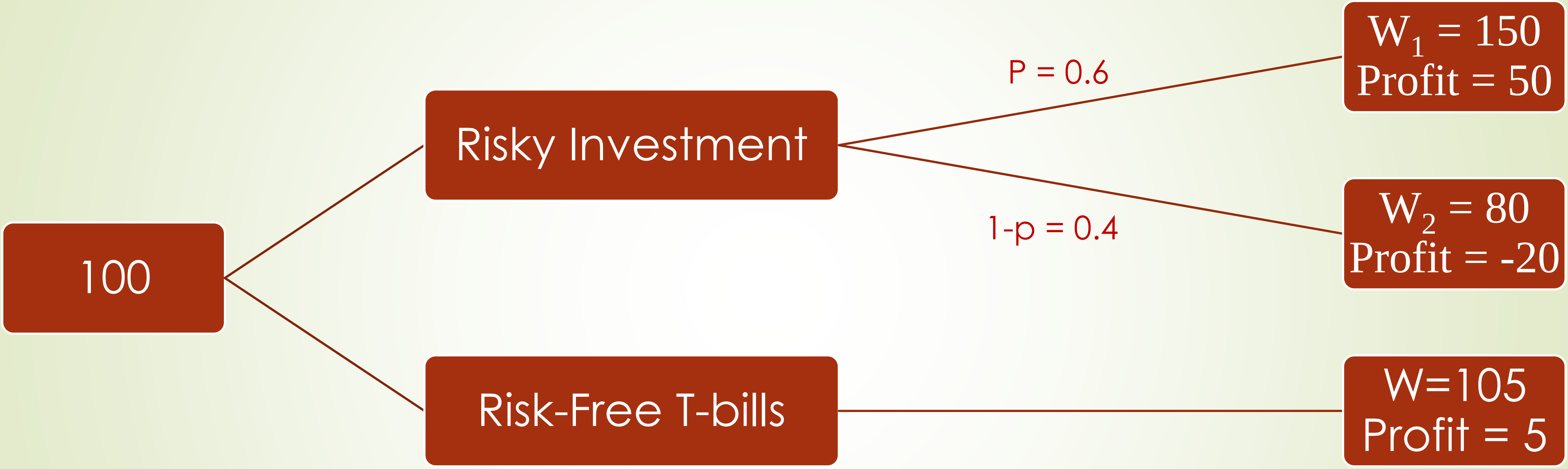
Questions

- **Q5**: Explain the notion of momentum crash.

- **Q6**: How would you mix two or more anomalies to establish a comprehensive trading strategy, e.g., momentum plus value?

Portfolio Optimization and Asset Classes

Risky and Risk-free Investments



If the two investments are mutually exclusive (you cannot mix), which one would you select?

Comparing the Investments

- Risky Investment:
  - $E(R) = pR_1 + (1\text{-}p)R_2 = 0.6 \times 0.50 + 0.4 \times (-0.2) = 0.22$
  - $\sigma^2 = p[R_1 - E(R)]^2 + (1\text{-}p)\,[R_2 - E(R)]^2 =$
    $0.6\,(0.50\text{-}0.22)^2 + 0.4(-0.2\text{-}0.22)^2 = 0.1176.$
    $\sigma = 0.3429$
- Risk-less Investment:
  - $E(R) = 0.05$ and $\sigma = 0$
  - *Risk Premium*: 22%-5% = 17%.

Characterizing Worldwide Investors

- Investors worldwide are, on average, risk averse.
- In the US and other countries, average rate of return on equities has exceeded T-Bill rate.
- Such a premium, often termed the *equity premium*, reflects a compensation for risky investments.
- The equity premium is typically computed as the return on a market-wide index, e.g., the S&P500, in excess of the return on the three-month T-Bill.

Risk Measures

- This lecture deals with the volatility of investment return as a risk measure.

- There are other well used measures termed down side risk measures, such as shortfall probability, downside beta, and VaR.

- Our investment framework is the mean variance methodology.

Risk Adjusted Return

$$U \;=\; E(R) - 0.5\, A\, \sigma^2$$

- U = risk adjusted (also termed certainty equivalent) return.
- E(R) = expected return.
- A = coefficient of risk aversion.
- σ^2 = variance of returns.

The idea here is to "penalize" expected return with the penalty factor depending on both volatility (supply side) and risk aversion (demand side).

Decision Rules

- The larger *A* the more risk averse the investor is.

- Very intuitive and heavily used among academic scholars as well as fund managers.

- Example: assume that a stock has $E(R) = 0.22$, $\sigma = 0.34$, and the return on T-bill is 5%.

- Suppose you can invest in the stock or T-bill. No mix! What will you do?

Risk Aversion and Investment Strategies

- It depends on the risk aversion level.
- Different risk aversion levels lead to different investment strategies.
- Recall, $U = 0.22 - 0.5\,A\,(0.34)^2$
- Hence:

A	Utility	Investment Decision
5	-6.90%	Buy T.bill (utility = 5%)
3	4.66%	Buy T.bill (utility = 5%)
1	16.22%	Buy stock (utility > 5%)

A Point of Indifference

- What is A such that investors are indifferent between the risky and risk-less investments?

- Solve:

$$0.05 = 0.22 - 0.5 \times A \times (0.34)^2$$

which yields

$$A = 2.9412.$$

- If A is smaller than 2.9412 invest in the risky asset. Otherwise, invest in the risk-free T.bill.

Mixing Risky and Risk-free Assets

- Suppose now that you want to invest in both the risky and risk-free assets.

- You seek the optimal combination (or mix) which maximizes the certainty equivalent rate of return

$$U = E(R) - 0.5\, A\, \sigma^2$$

- What is the optimal mix as a function of the expected return and volatility of the risky asset, the risk-free rate, and the risk aversion level A?

Risk Aversion Leads to Diversification

- A sensible strategy for a risk averse agent would be to diversify funds across several securities.
- A risk neutral investor would simply pick the highest expected return investment. No diversification!!
- I will give examples displaying how diversification reduces risk.
- To study the optimal diversification, or the optimal portfolio a risk-averse investor would choose, you should know how to compute expected return and volatility of portfolios.
- This is what we do next.

Diversification and Asset Allocation

- The role and impact of diversification were formally introduced in the early 1950s by professor Harry Markowitz.
- Based on his work, we will look at how diversification works, and how we can create efficiently diversified portfolio.
- An efficient portfolio is one having the highest expected return for a certain level of risk.
- OR an efficient portfolio is one having the lowest risk for a certain level of expected return.

Portfolio of Risky Assets

- **Portfolios** are group of assets such as stocks and bonds held by an investor.
- In the analysis that follows we will master the following key concepts that establish the notion of diversification:
  - portfolio weights.
  - portfolio expected return and volatility.
  - Correlation.

Computing Expected Return: An Example

- You invest \$4,000 in domestic equities, \$3,000 in real estate, \$2,000 in commodities, and \$1,000 in foreign bonds.
- Portfolio weights are 0.4, 0.3, 0.2, and 0.1.
- Expected returns are 12%, 10%, 9%, and 8%, respectively.
- What is the portfolio expected rate of return?
- Solve:

$$0.4\times 12\% + 0.3\times 10\% + 0.2\times 9\% + 0.1\times 8\% = 10.4\%.$$

What about Volatility?

- More challenging!
- We will start with a relatively simple case of a two-security portfolio.
- We will then generalize the concept to portfolios consisting of more than two securities.

Volatility of a Two-Security Portfolio

- When two risky assets are combined to form a portfolio with weights w_A and $w_B = 1 - w_A$, the portfolio variance is computed as:

$$\sigma_p^2 = w_A^2 \sigma_A^2 + (1 - w_A)^2 \sigma_B^2 + 2 w_A (1 - w_A) \sigma_A \sigma_B \rho_{AB}$$

Example

- The volatilities of JP Morgan and Goldman Sachs are 16% and 20%, respectively. The covariance is 0.01.

- What is the variance of a portfolio that invests 75% in JP and 25% in Goldman:

$$\sigma_p^2 = 0.75^2 \times 0.16^2 + 0.25^2 \times 0.2^2 \ + 2 \times 0.75 \ \times 0.25 \times 0.01 = 0.0207$$

- The portfolio volatility is 14.37% -- the square root of the variance -- which is lower than 16% (JP) or 20% (Goldman).

- This is a diversification benefit.

Why does Diversification Work? Correlation

- *Correlation*: the tendency of the returns on two assets to move together.

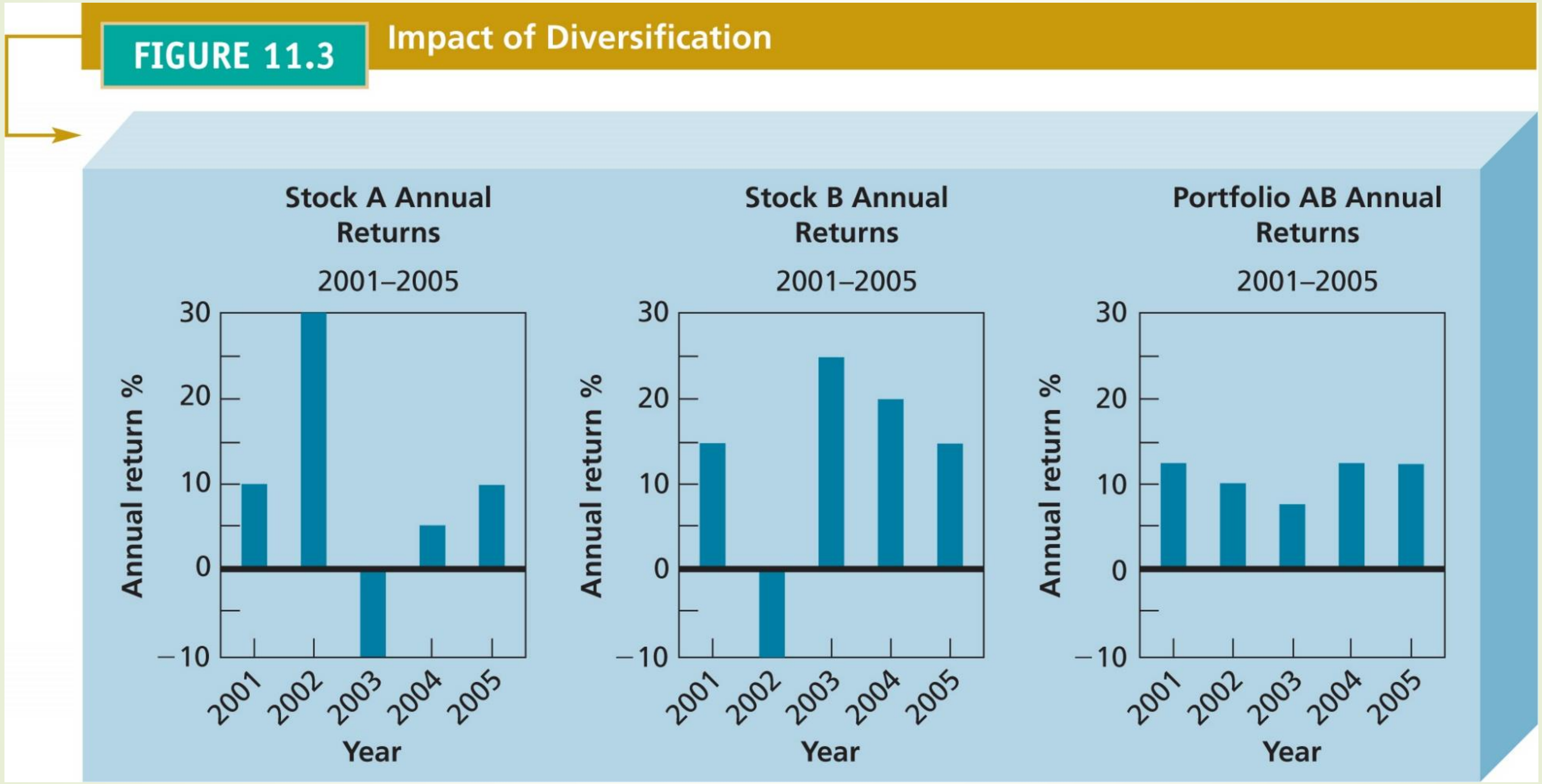
  - *Positively* correlated assets tend to move up and down together, while *negatively* correlated assets tend to move in opposite directions.

  - Imperfect correlation is why diversification reduces portfolio risk.

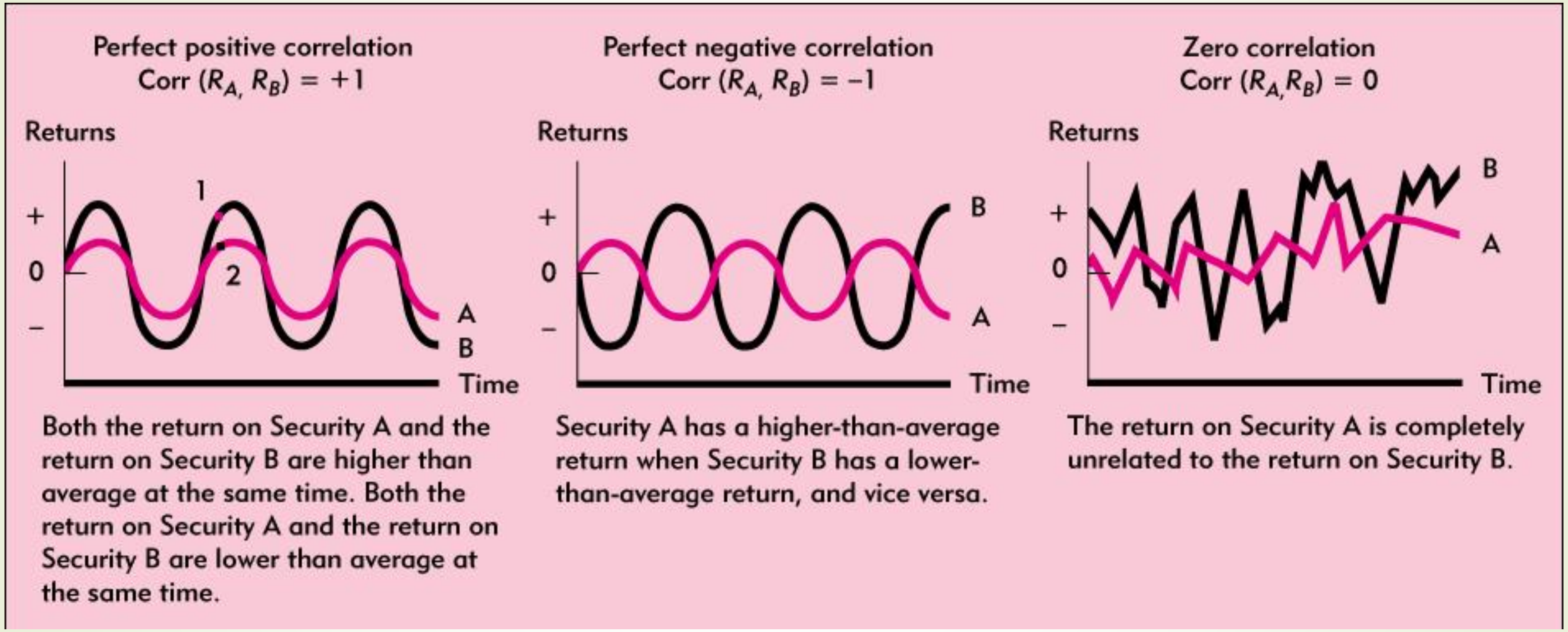
Why does Diversification Work?

Year	Stock A (%)	Stock B (%)	Portfolio AB (%)
2001	10	15	12.5
2002	30	-10	10
2003	-10	25	7.5
2004	5	20	12.5
2005	10	15	12.5
Average return	9	13	11
Standard deviation	14.3	13.5	2.2

Why does Diversification Work?



More about Correlation



The Optimal Mix of JPM and GS

- What is the investment in JP Morgan that minimizes the portfolio volatility?
  - Answer: 65.79% - formulas are coming up!!

- What is the portfolio volatility?
  - Answer: 14.23%, even better.

GMVP

- In risk-management applications, investors typically care only about reducing risk.
- Such investors seek the minimum-variance combination.
- Ex ante, GMVP delivers the lowest payoff among all efficient portfolios - but ex post it performs well.
- How can we find the minimum-variance combination?
- When you can invest only in two-risky securities – it is relatively easy to find the GMVP

Finding Minimum-Variance Combinations

$$Sec\ 1\ E(r_1) = 0.10 \quad \sigma_1 = 0.15$$

$$\rho_{12} = 0.2$$

$$Sec\ 2\ E(r_2) = 0.14 \quad \sigma_2 = 0.20$$

$$w_1 = \frac{\sigma_2^2 - Cov(r_1 r_2)}{\sigma_1^2 + \sigma_2^2 - 2\,Cov(r_1 r_2)}$$

$$w_2 = (1 - w_1)$$

Minimum-Variance Combination: ρ = 0.2

$$w_1 = \frac{(0.2)^2 - (0.2)(0.15)(0.2)}{(0.15)^2 + (0.2)^2 - 2(0.2)(0.15)(0.2)}$$

$$w_1 = 0.6733$$

$$w_2 = (1 - 0.6733) = 0.3267$$

GMVP in more complex cases

- What if you have more than two security classes?
- The obtained expression for the GMVP is still analytical but complex.
- What if you also account for portfolio constraints?
- Use Solver installed in Excel but first you need to study how to properly set up the optimization problem.
- Extending to a Three-Security Portfolio:

$$\mu_p = w_1\mu_1 + w_2\mu_2 + w_3\mu_3$$

$$\begin{aligned}\sigma_p^2 = {} & w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\,\sigma_3^2 \\ & + 2w_1w_2\,cov(r_1, r_2) \\ & + 2w_1w_3\,cov(r_1, r_3) \\ & + 2w_2w_3\,cov(r_2, r_3)\end{aligned}$$

The Case of Multiple Asset Classes

- Computing volatility in the presence of multiple asset classes is challenging.
- Both HMC case studies are about multiple asset classes.
- Our focus here is a four-security portfolio.
- Generalizations follow the same vein.

Computing Volatility

- The volatilities of domestic equities, real estate, commodities, and foreign bonds are 18%, 17%, 16%, and 17%, respectively.
- Investment weights are 0.4, 0.3, 0.2, and 0.1
- Correlations are

1	0.8	0.6	0.5
0.8	1	0.7	0.8
0.6	0.7	1	0.4
0.5	0.8	0.4	1

Array # 1: Product of Weights

You need to create three arrays.

- Array # 1 is the product of portfolio weights:

0.4×0.4	0.4 ×0.3	0.4 ×0.2	0.4 ×0.1
0.3×0.4	0.3 ×0.3	0.3 ×0.2	0.3 ×0.1
0.2×0.4	0.2 ×0.3	0.2 ×0.2	0.2 ×0.1
0.1×0.4	0.1 ×0.3	0.1 ×0.2	0.1 ×0.1

Array # 2 : The Covariance Matrix

- Array # 2 is the covariance matrix:

0.18×0.18	0.8×0.18×0.17	0.6×0.18×0.16	0.5×0.18×0.17
0.8×0.18×0.17	0.17×0.17	0.7×0.17×0.16	0.8×0.17×0.17
0.6×0.18×0.16	0.7×0.17×0.16	0.16×0.16	0.4×0.16×0.17
0.5×0.18×0.17	0.8×0.17×0.17	0.4×0.16×0.17	0.17×0.17

Array # 3 : Computing Volatility

- Array # 3 is obtained by multiplying array 1 by array 2, element by element:

0.0052	0.0029	0.0014	0.0006
0.0029	0.0026	0.0011	0.0007
0.0014	0.0011	0.0010	0.0002
0.0006	0.0007	0.0002	0.0003

Benefits from Diversification

- The portfolio variance follows by summing over all the elements in array # 3, which gives 0.0229.
- The portfolio volatility is 15.13% -- the square root of 0.0229.
- *Diversification gain*: the volatility of single securities ranges between 16% and18%, whereas the portfolio volatility is smaller.
- Once again, I did not even come up with the lowest volatility portfolio (GMVP).
- What is the GMVP? Coming soon!

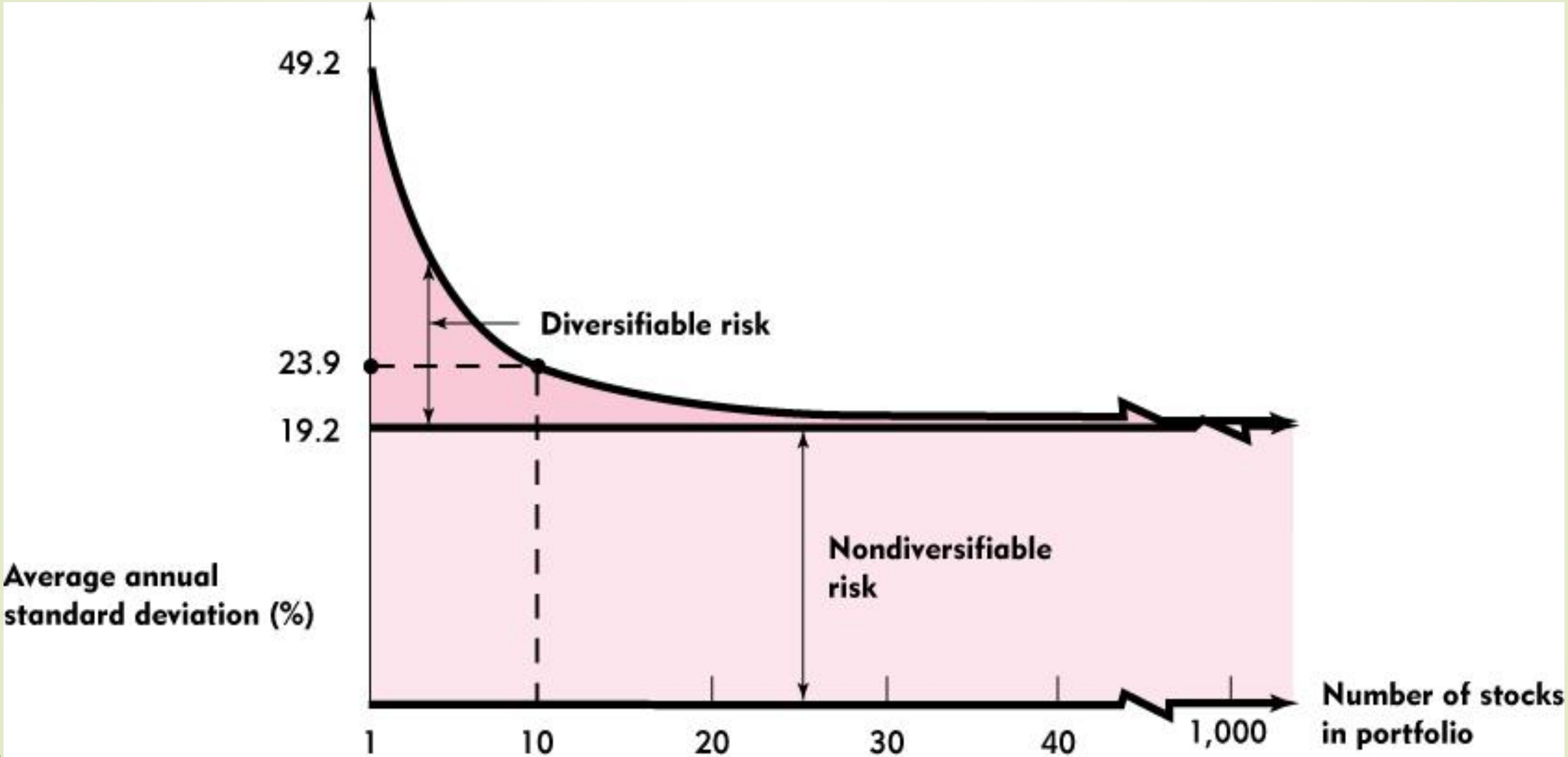
The Contribution of a Single Security to the Overall Risk

- Column number 1 denotes the contribution of domestic equities to the total risk.

- Example: summing up the components of column one yields 0.0101

- The relative contribution of domestic equities to the total risk is 0.0101 / 0.0229 = 44.11%.

The Principle of Diversification

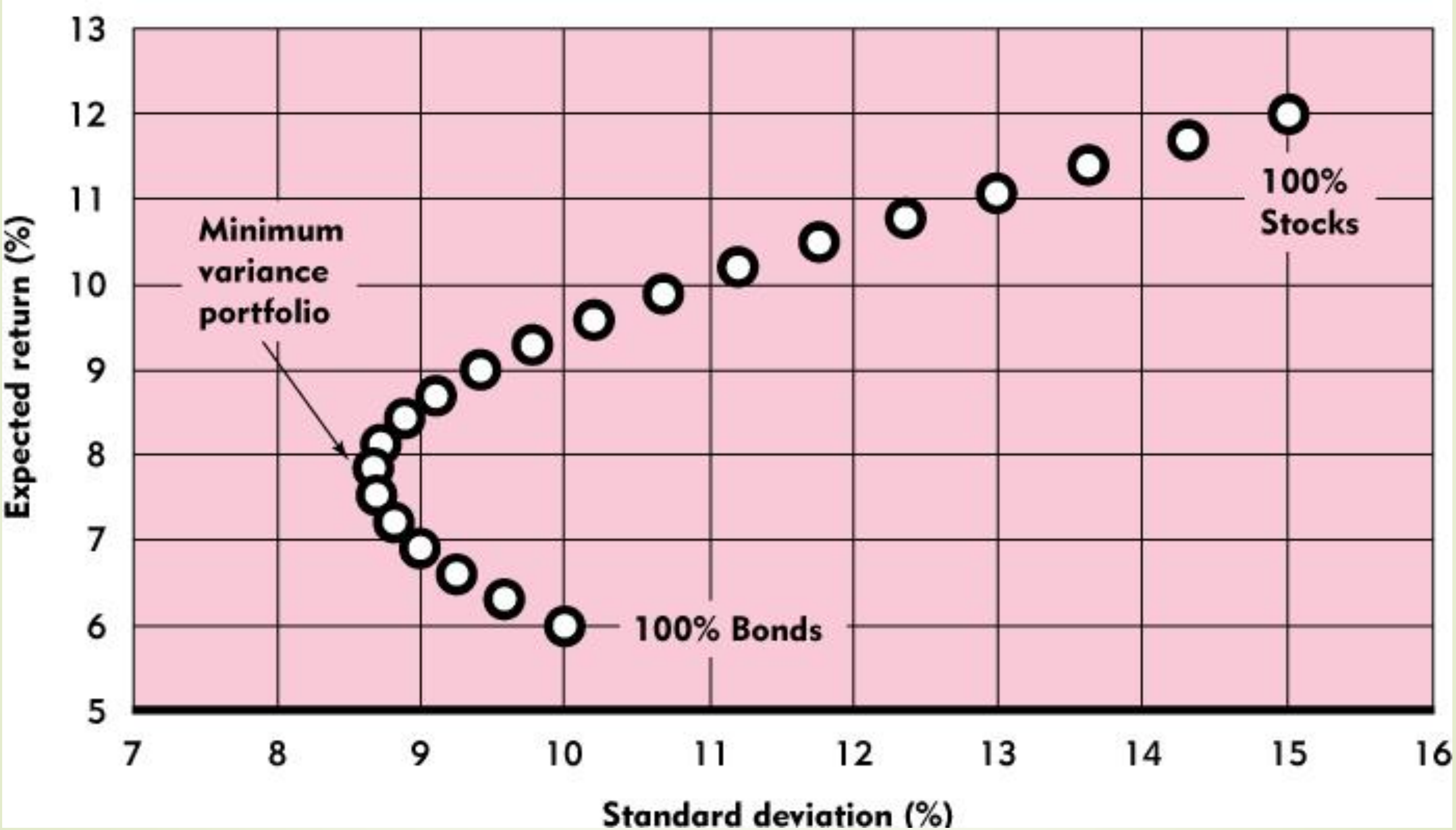


Combining Stock and Bond

Risk and Return with Stocks and Bonds

Portfolio Weights			
Stocks	Bonds	Expected Return	Standard Deviation
1.00	.00	12.00%	15.00%
.95	.05	11.70	14.31
.90	.10	11.40	13.64
.85	.15	11.10	12.99
.80	.20	10.80	12.36
.75	.25	10.50	11.77
.70	.30	10.20	11.20
.65	.35	9.90	10.68
.60	.40	9.60	10.28
.55	.45	9.30	9.78
.50	.50	9.00	9.42
.45	.55	8.70	9.12
.40	.60	8.40	8.90
.35	.65	8.10	8.75
.30	.70	7.80	8.69
.25	.75	7.50	8.71
.20	.80	7.20	8.82
.15	.85	6.90	9.01
.10	.90	6.60	9.27
.05	.95	6.30	9.60
.00	1.00	6.00	10.00

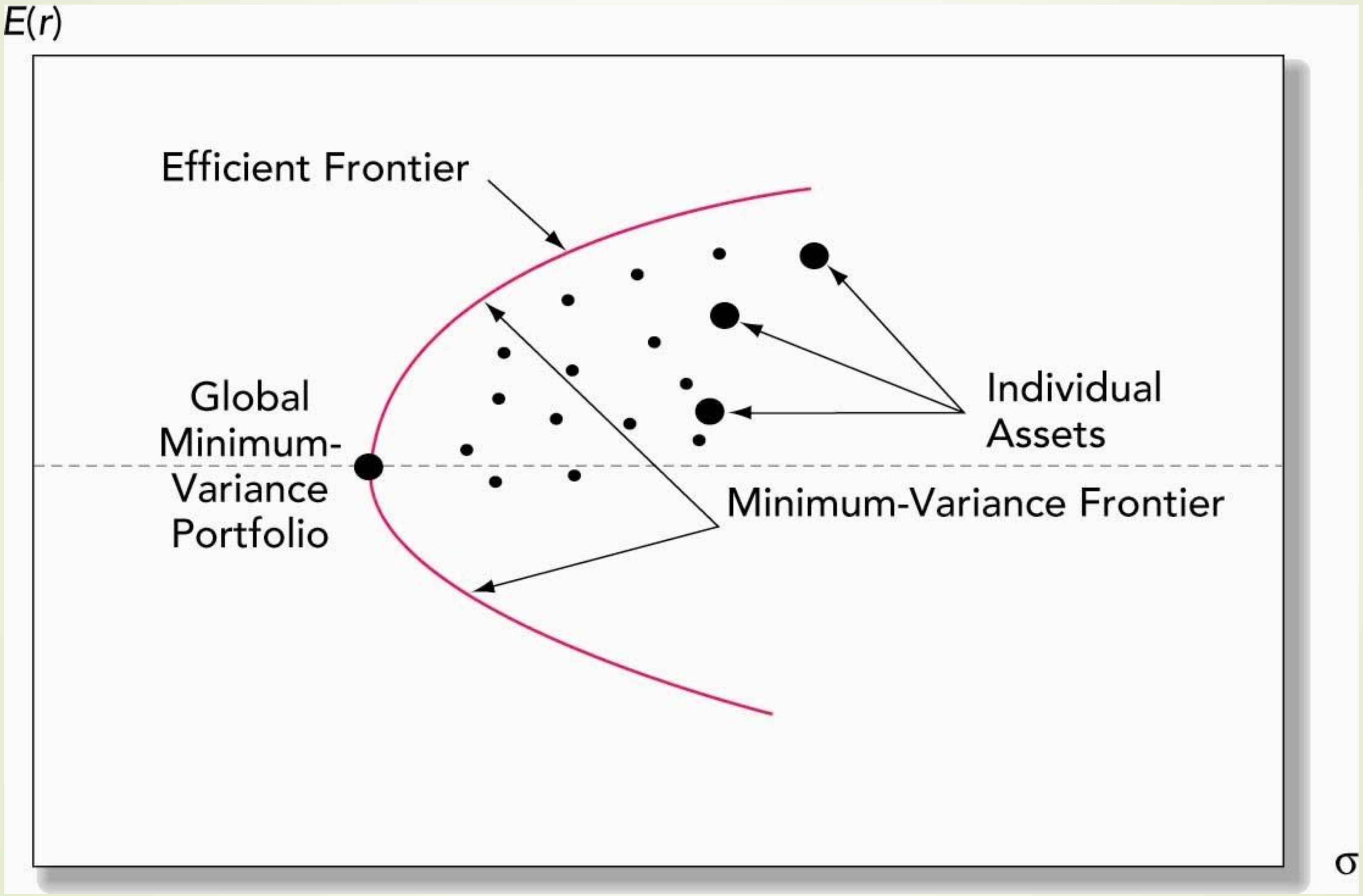
Efficient Frontier for Stock and Bond



The Efficient Frontier for Multiple Risky Assets

- The optimal combinations result in the lowest level of risk for a given expected return.
- The optimal trade-off is described as the efficient frontier of risky securities.
- An optimizing mean-variance investor would pick only portfolios that lie on the efficient frontier.
- Exhibit 12 in the HMC case displays 22 different portfolios along the efficient frontier. Every portfolio is a combination of the 12 asset classes.

The Minimum-Variance Frontier of Risky Assets



Do Investors Hold a Well-Diversified Portfolio?

- A research paper by Campbell, Lettau, Malkiel, and Xu (JF 2001) suggest that the number of randomly selected stocks needed to achieve relatively complete portfolio diversification is about 50.
- This figure could be higher or lower during different periods.
- Goetzmann and Kumar (Review of Finance 2008) show that, based on a sample of 62,000 household investors:
- More than 25% of the investor portfolios contain only one stock.
- Over half of the portfolios contain no more than three stocks.
- Less than 10% of the portfolios contain more than 10 stocks.

- Obviously, US investors under-diversify.

End-of-Chapter Questions

- **Q1**: A portfolio has an expected rate of return of 0.12 and a standard deviation of s=0.13. The risk-free rate is 6.085%. An investor has the following utility function: $U = E(r) - (A/2)s^2$. Which value of A makes this investor indifferent between the risky portfolio and the risk-free asset?

- **Q2**: There are four investable portfolios: W(15,36), X(12,15), Y(5,7), and Z(9,21) – which one cannot lie on the efficient frontier? Are the other portfolios essentially efficient?

Questions

- **Q3**: The Sharpe ratios and Jensen's alphas of five actively managed mutual funds and the S&P 500 index are given by

Fund	Sharpe ratio	Jensen's alpha
A	0.40	0.01
B	0.50	0.02
C	0.20	0.01
D	0.10	-0.02
E	0.25	-0.01
S&P 500	0.30	0.00

You apply the following decision rule: an outperforming fund is one that dominates along both the Sharpe ratio and Jensen's alpha criteria. Based upon that decision rule, which of the funds outperform the S&P index.

Questions

- **Q4**: A portfolio consists of the following two securities

	A	B
Expected Return	16%	10%
Standard Deviation	25%	15%
Portfolio Market Value	$25,000	$15,000
Correlation	0.3	
Risk Free Rate	4%	

What is the Sharpe ratio of the portfolio?

- **Q5**: Stocks offer Ex=18% and Vol=22%. Gold offers Ex=10% and Vol=30%. Would you invest in gold? Use plot to explain!

Questions

- **Q6**: You have the following information

	A	B
Expected Return	16%	10%
Standard Deviation	25%	15%
Correlation	0.3	
Risk Free Rate	4%	

You invest in A, B, and the riskfree asset in two steps. First you generate the GMVP from stocks A and B. You then optimally mix the GMVP with the riskfree asset attempting to maximize the certainty equivalent rate of return when the risk aversion parameter is two. What are the ultimate fractions invested in A, B, and the riskfree asset.

Derivatives Securities

Overview

- Option terminology.
- Call and put options.
- Forward and futures contracts.
- Speculation with derivatives.
- Hedging with derivatives.
- The Put-Call Parity.
- The Binomial Tree.
- The B&S formula for pricing Call, Put, and Exotic Options.

What are Derivatives?

- **Primary assets**: Securities sold by firms or government to raise money (stocks and bonds) as well as stock indexes (e.g., S&P, Nikkei), exchange rates ($ versus £), and commodities (e.g., gold, coffee).

- **Derivative assets**: Options, forward and futures contracts, Eurodollars, CDS, swaptions, etc. These financial assets are derived from existing primary assets.

Option Terminologyf

- Buy = Long = Hold
- Sell = Short = Write
- Call - option to buy underlying asset
- Put - option to sell underlying asset
- So we have: buy call, buy put, sell call, sell put
- Key Elements:
  - Exercise or Strike Price
  - Maturity or Expiration
  - Premium or Price
  - Zero Sum Game

Definition and Terminology

- A call option gives the owner the right **but not the obligation** to buy the underlying asset at a predetermined price during a predetermined time period. Note the right belongs to the buyer not the seller.
- Strike (or exercise) price: the amount paid by the option buyer for the asset if he/she decides to exercise.
- Exercise: the act of paying the strike price to buy the asset.
- Expiration: the date by which the option must be exercised or become worthless.
- Exercise style: specifies when the option can be exercised.
  - European-style: can be exercised only at expiration date.
  - American-style: can be exercised at any time before expiration.
  - Bermudan-style: Can be exercised during specified periods (e.g., on the first day of each month. Bermuda is located between the US and Europe.

Examples

Example: S&R (special & rich) index

- Today: **call buyer** acquires the right to pay $1,020 in six months for the index, but is not obligated to do so
- In six months at contract expiration: if spot price is
  - $1,100, call buyer's payoff = $1,100 – $1,020 = $80
  - $ 900, call buyer walks away, buyer's payoff = $0
- Today: **call seller** is obligated to sell the index for $1,020 in six months, if asked to do so
- In six months at contract expiration: if spot price is
  - $1,100, call seller's payoff = $1,020 – $1,100 = ($80)
  - $900, call buyer walks away, seller's payoff = $0

Payoff/Profit of a Purchased Call

- Payoff = Max [0, spot price at expiration–strike price]
- Profit = Payoff – future value of option premium

Example:

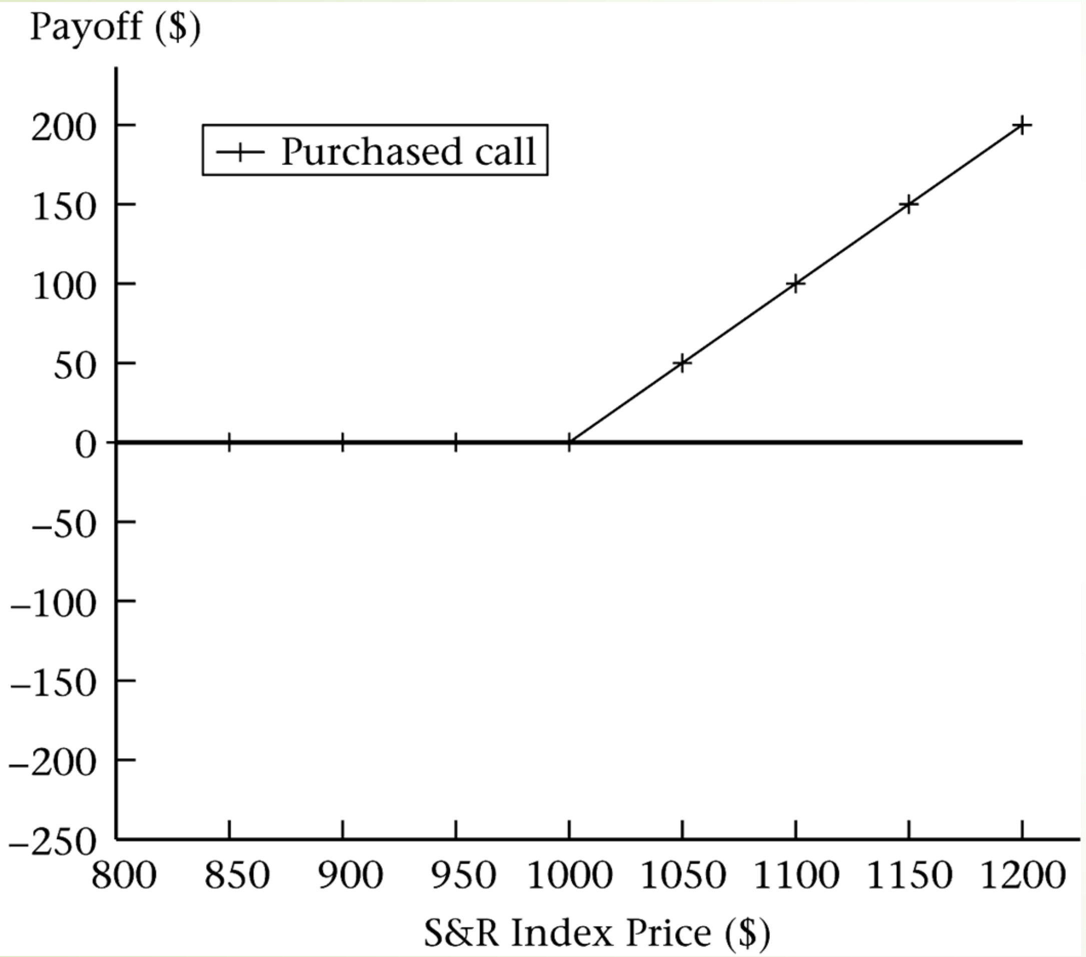
S&R Index 6-month Call Option

Strike price = \$1,000, Premium = \$93.81, 6-month, risk-free rate = 2%

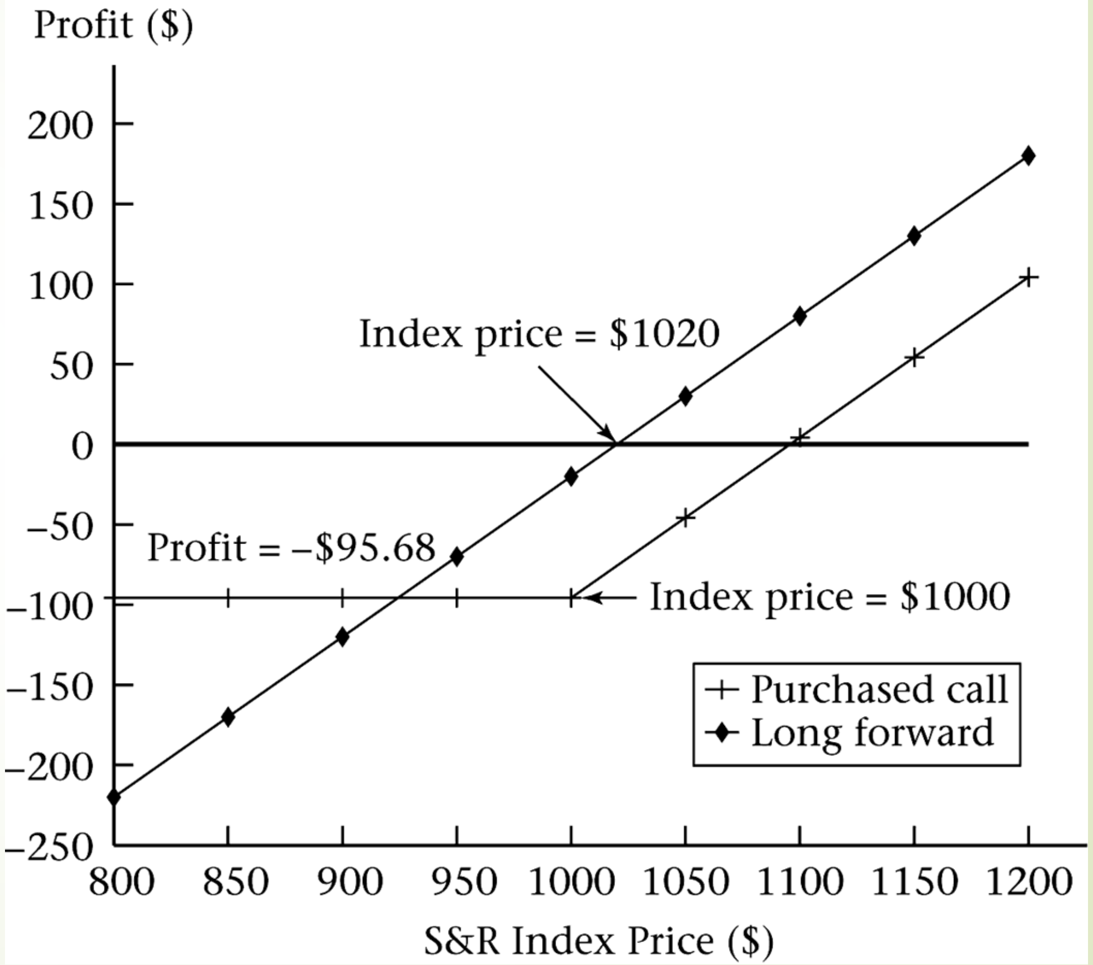
- If index value in six months = \$1100
  - Payoff = max [0, \$1,100 – \$1,000] = \$100
  - Profit = \$100 – (\$93.81 × 1.02) = \$4.32
- If index value in six months = \$900
  - Payoff = max [0, \$900 – \$1,000] = \$0
  - Profit = \$0 – (\$93.81 × 1.02) = – \$95.68

Diagrams for Purchased Call

Payoff at expiration



Profit at expiration



Payoff/Profit of a Written Call

- Payoff = – max [0, spot price at expiration–strike price]
- Profit = Payoff + future value of option premium

Example:

S&R Index 6-month Call Option

Strike price = $1,000, Premium = $93.81, 6-month, risk-free rate = 2%

- If index value in six months = $1100
  - Payoff = – max [0, $1,100 – $1,000] = – $100
  - Profit = – $100 + ($93.81 × 1.02) = – $4.32
- If index value in six months = $900
  - Payoff = – max [0, $900 – $1,000] = $0
  - Profit = $0 + ($93.81 × 1.02) = $95.68

Put Options

- A put option gives the owner the right but not the obligation to sell the underlying asset at a predetermined price during a predetermined time period.
- The seller of a put option is obligated to buy if asked.

- Payoff/profit of a purchased (i.e., long) put
  - Payoff = max [0, strike price – spot price at expiration]
  - Profit = Payoff – future value of option premium
- Payoff/profit of a written (i.e., short) put
  - Payoff = – max [0, strike price – spot price at expiration]
  - Profit = Payoff + future value of option premium

Put Option Examples

S&R Index 6-month Put Option

Strike price = $1,000, Premium = $74.20, 6-month, risk-free rate = 2%

- If index value in six months = $1100
  - Payoff = max [0, $1,000 – $1,100] = $0
  - Profit = $0 – ($74.20 × 1.02) = – $75.68
- If index value in six months = $900
  - Payoff = max [0, $1,000 – $900] = $100
  - Profit = $100 – ($74.20 × 1.02) = $24.32

A Few Items to Note

- A call option becomes more profitable when the underlying asset appreciates in value.

- A put option becomes more profitable when the underlying asset depreciates in value.

- Moneyness is an important concept in option trading.

"Moneyness"

- **In the Money** - exercise of the option would be profitable.
  - Call: market price > exercise price (denoted by K or X).
  - Put: exercise price > market price.

- **Out of the Money** - exercise of the option would not be profitable.
  - Call: market price < exercise price.
  - Put: exercise price < market price.

- **At the Money** - exercise price and market price are equal.

Options on IBM

PRICES AT CLOSE MARCH 23, 2006

I B M (IBM) **Underlying stock price: 83.20**

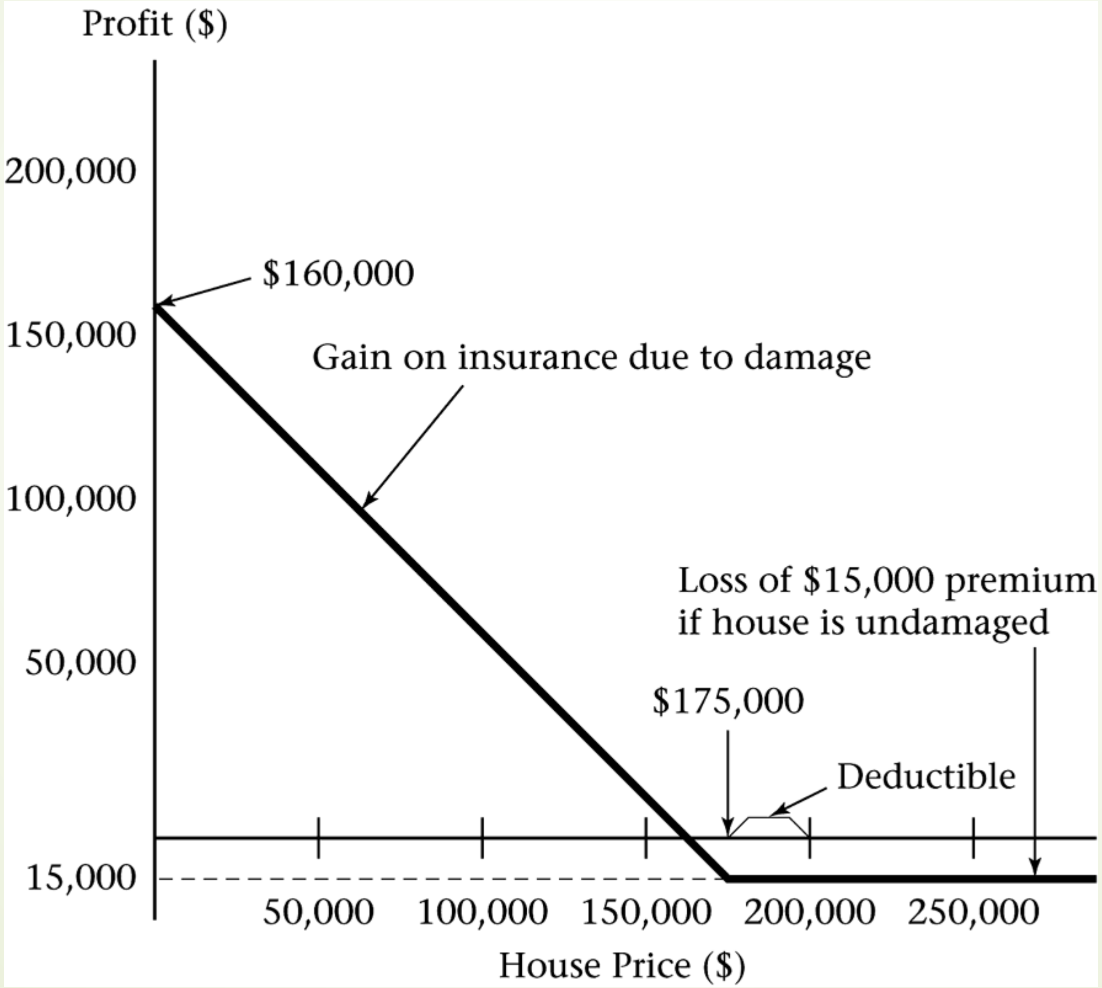
		Call			Put		
Expiration	**Strike**	**Last**	**Volume**	**Open Interest**	**Last**	**Volume**	**Open Interest**
Apr	75.00	8.50	201	2568	0.10	27	19877
May	75.00	...	...	239	0.20	29	170
Jul	75.00	10.10	1	962	0.50	30	9616
Oct	75.00	11.10	1	378	1.10	56	541
Apr	80.00	4.10	1390	19671	0.55	3378	32086
May	80.00	4.40	174	215	0.75	1052	513
Jul	80.00	5.50	57	4357	1.44	234	10156
Oct	80.00	7.30	5	892	2.20	79	1114
Apr	85.00	0.95	2221	42456	2.45	1548	16330
May	85.00	1.35	331	1300	2.90	676	959
Jul	85.00	2.59	570	19451	3.50	103	7963
Oct	85.00	4.10	9	1073	...	...	804
Apr	90.00	0.15	989	21447	6.80	146	587
May	90.00	0.25	7	348	6.80	26	89
Jul	90.00	0.85	353	17257	7.00	670	792
Oct	90.00	2.15	2516	4587	7.40	25	194

Homeowner's Insurance is a Put Option

- You own a house that costs $200,000.
- You buy a $15,000 insurance policy.
- The deductible amount is $25,000.
- Let us graph the profit from this contact

Options and Insurance

Homeowner's insurance as a put option.



Call Options are also Insurance

- Banks and insurance companies offer investment products that allow investors to benefit from a rise in a stock index and provide a guaranteed return if the market falls.

- The equity linked CD provides a zero return if the index falls (refund of initial investment) and a return linked to the index if the index rises.

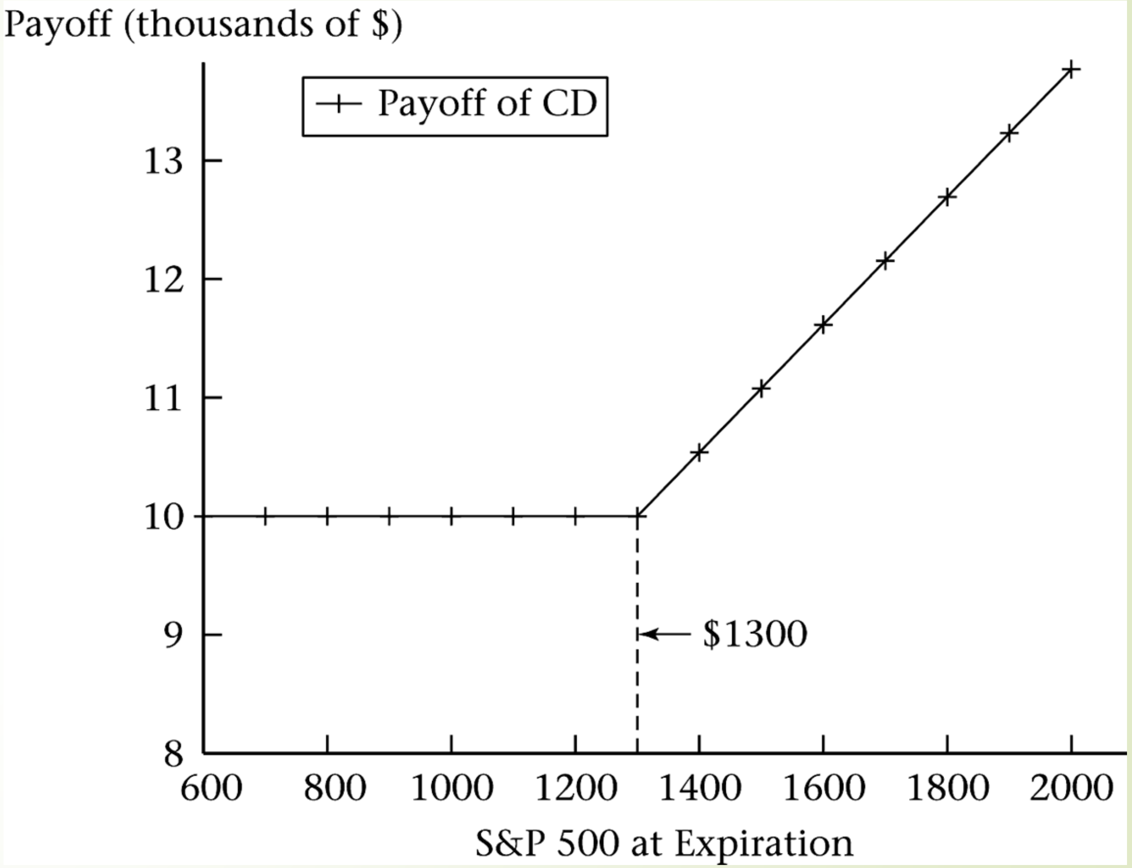
Structure: Equity Linked CDs

The 5.5-year CD promises to repay initial invested amount plus 70% of the gain in S&P500 index.

- Assume $10,000 is invested when S&P 500 = 1300
- Final payoff =

$$\$10{,}000 \times \left(1 + 0.7 \times \max\left[0, \frac{S_{final}}{1300} - 1\right]\right)$$

- Where S_{final} = the value of the S&P500 that will be in 5.5 years



The Economic Value of the Equity Linked CD

- We paid \$10,000 and we get \$10,000 in 5.5 years plus some extra amount if the S&P500 index level exceeds 1300.
- That payoff structure is equivalent to buying a zero coupon bond and x call options.
- Why? Assuming that the annual effective rate is 6%, the present value of \$10,000 to be received in 5.5 years is \$7,258.
- Thus, we practically paid \$7,258 for a zero coupon bond and \$2,742 for x call options.
- What is x? And what is the implied value of one call option?
- What is the fraction of the gain in the S&P500 index we should get if the current value of one call option is \$450?

Forward Contract

- A **forward contract** is an agreement made **today** between a buyer and a seller who are **obligated** to complete a transaction at a pre-specified date in the future.

- The buyer and the seller know each other. The negotiation process leads to **customized** agreements: What to trade; Where to trade; When to trade; How much to trade?

Futures Contract

- A **Futures contract** is an agreement made **today** between a buyer and a seller who are **obligated** to complete a transaction at a pre-specified date in the future.
- The buyer and the seller **do not** know each other. The "negotiation" occurs in the fast-paced frenzy of a futures pit.
- The terms of a futures contract are **standardized**. The contract specifies what to trade; where to trade; When to trade; How much to trade; what quality of good to trade.

Understanding Price Quotes

THE WALL STREET JOURNAL.

FUTURES

TUESDAY, JULY 19, 2005

Interest Rate Futures

	OPEN	HIGH	LOW	SETTLE	CHG	LIFETIME HIGH	LIFETIME LOW	OPEN INT
Treasury Bonds (CBT)-$100,000; pts 32nds of 100%								
Sept	116-15	116-20	115-14	115-22	–23	119-23	108-15	560,897
Dec	115-28	116-05	115-03	115-09	–23	119-07	108-23	15,697

Est vol 162,922; vol Fri 198,479; open int 576,644, +686.

Payoff on a Futures Contract

- Payoff for a contract is its value at expiration.
- Payoff for
  - Long forward = Spot price at expiration – Forward price
  - Short forward = Forward price – Spot price at expiration

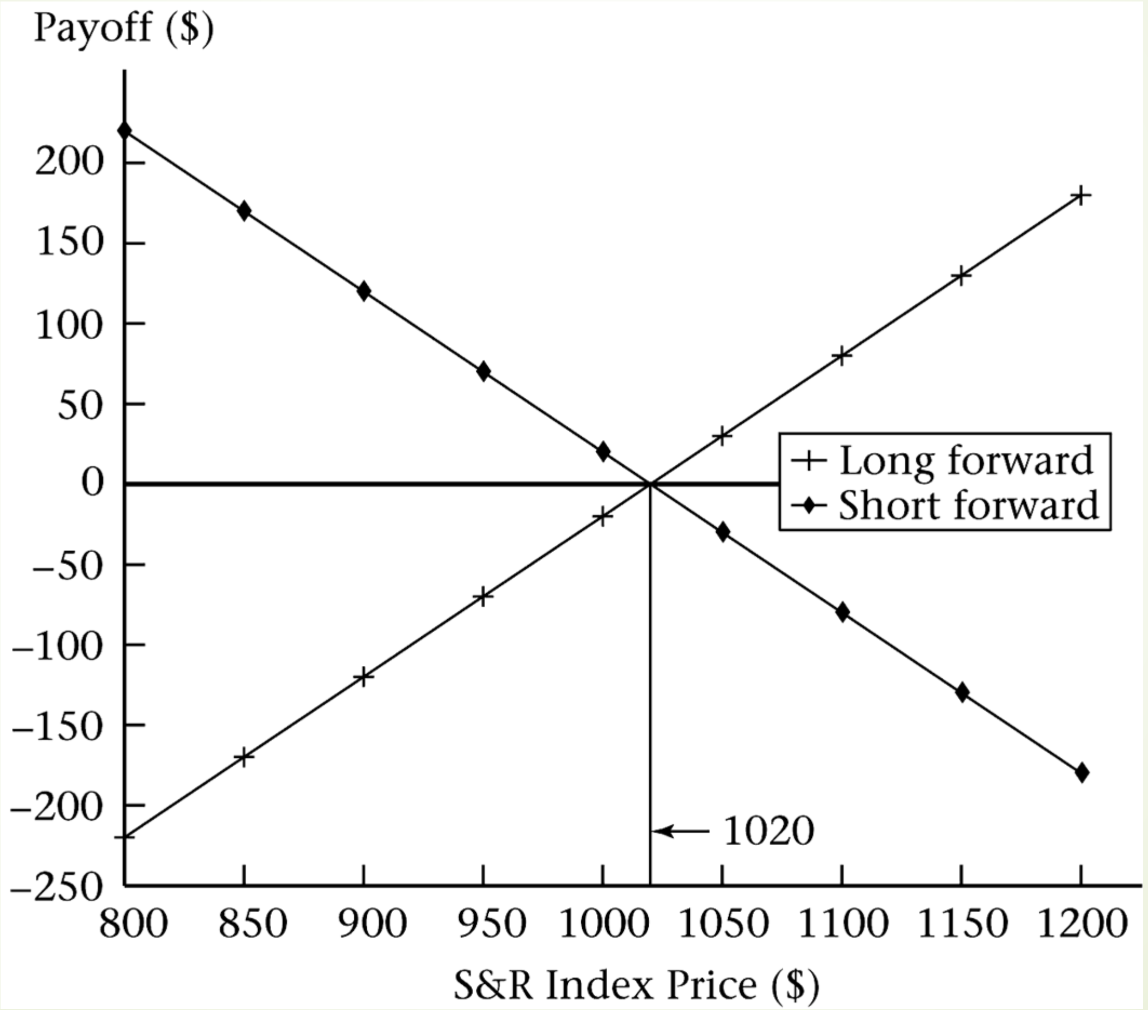
<u>Example</u>: S&R index:

- Today: Spot price = $1,000, 6-month forward price = $1,020

- In six months at contract expiration: Spot price = $1,050
  - Long position payoff = $1,050 – $1,020 = $30
  - Short position payoff = $1,020 – $1,050 = ($30)

Payoff Diagram for Futures

Long and short forward positions on the S&R 500 index.



Example: Speculating in Gold Futures, Long

- You believe the price of gold will go **up**. So,
  - You go **long** 100 futures contract that expires in 3 months.
  - The futures price today is **$400** per ounce.
  - Assume interest rate is zero.
  - There are 100 ounces of gold in each futures contract.
- Your "position value" is: $400 × 100 × 100 = $4,000,000
- Suppose your belief is correct, and the price of gold is $420 when the futures contract expires.
- Your “position value” is now: $420 × 100 × 100 = $4,200,000
- Your “long” speculation has resulted in a **gain** of **$200,000**

Example: Speculating in Gold Futures, Short

- You believe the price of gold will go **down**. So,
  - You go **short** 100 futures contract that expires in 3 months.
  - The futures price today is **$400** per ounce.
  - Assume interest rate is zero.
  - There are 100 ounces of gold in each futures contract.
- Your "position value" is: $400 × 100 × 100 = $4,000,000

- Suppose your belief is correct, and the price of gold is $370 when the futures contract expires.
- Your "position value" is now: $370 × 100 × 100 = $3,700,000

- Your "short" speculation has resulted in a **gain** of **$300,000**

Risk management: The Producer's Perspective

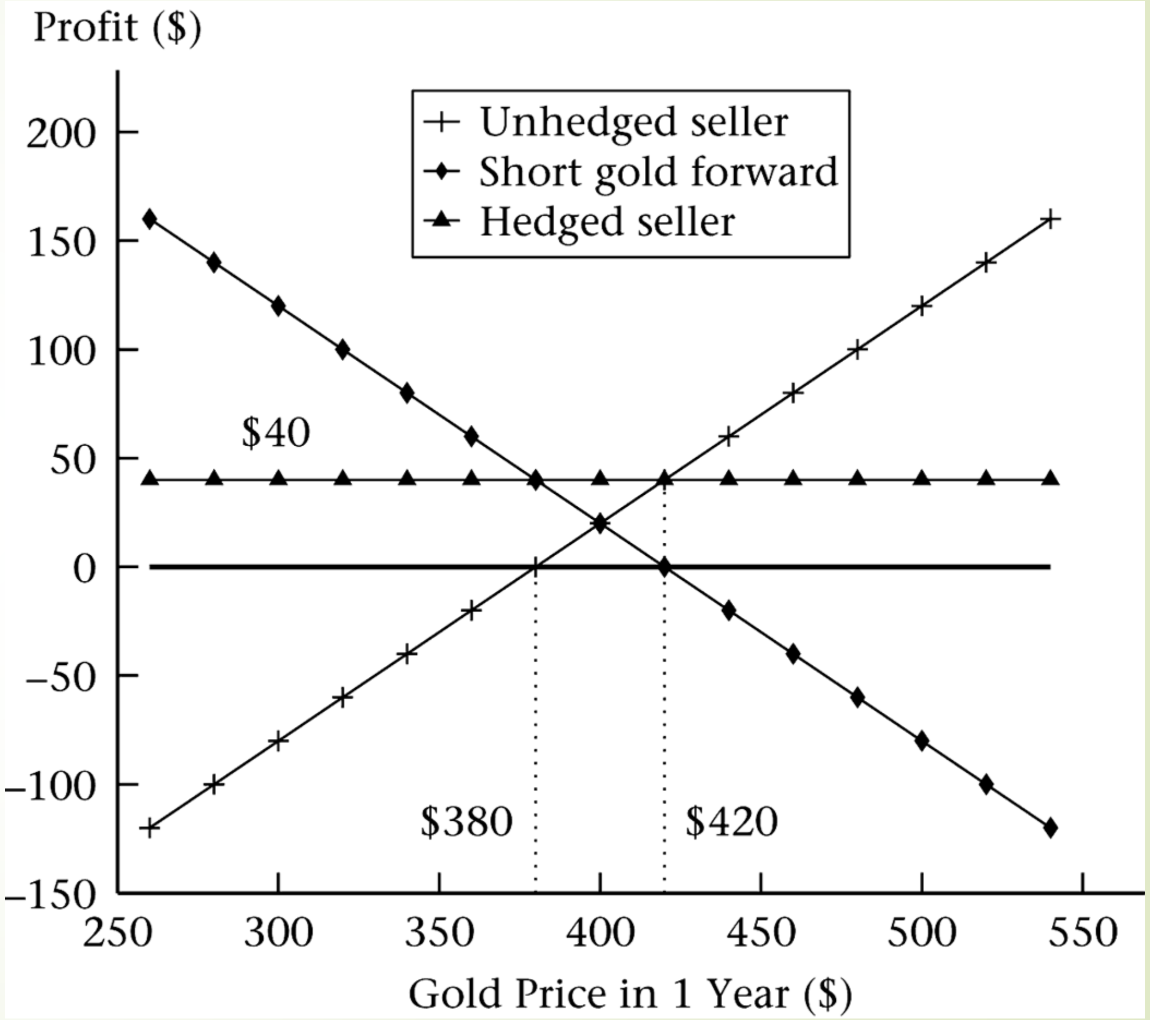
- We can also use futures contracts for hedging.
- A producer selling a risky commodity has an inherent long position in this commodity.
- When the price of the commodity increases, the profit typically increases.
- Common strategies to hedge profit:
  - Selling forward.
  - Buying puts.
  - Selling Calls.

Producer: Hedging With a Forward Contract

A short forward contract allows a producer to lock in a price for his output.

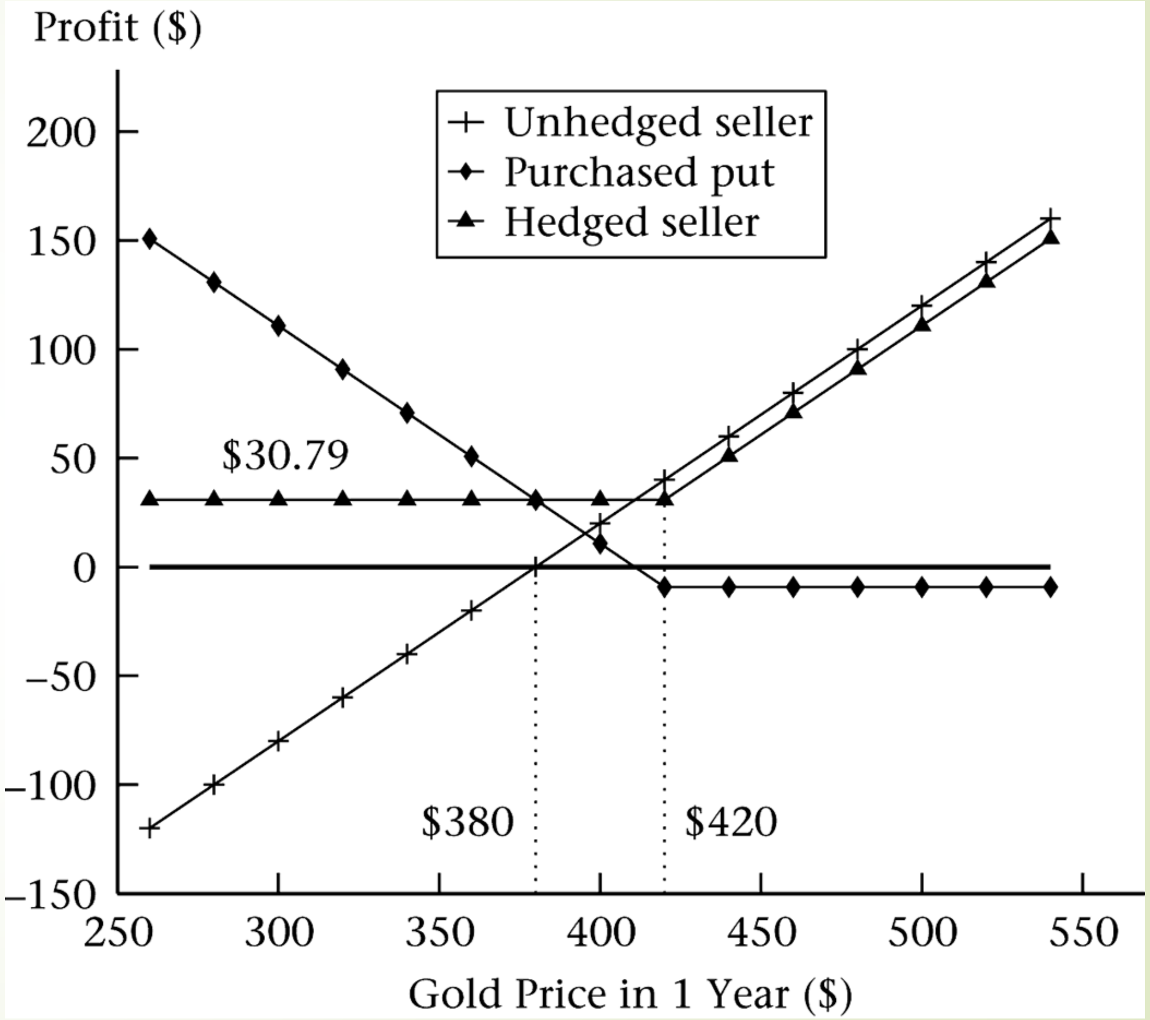
<u>Example</u>:

A gold-mining firm enters into a short forward contract, agreeing to sell gold at a price of $420/oz. in 1 year. The cost of production is $380.



Producer: Hedging With a Put Option

Buying a put option allows a producer to have higher profits at high output prices, while providing a floor on the price.



Example:

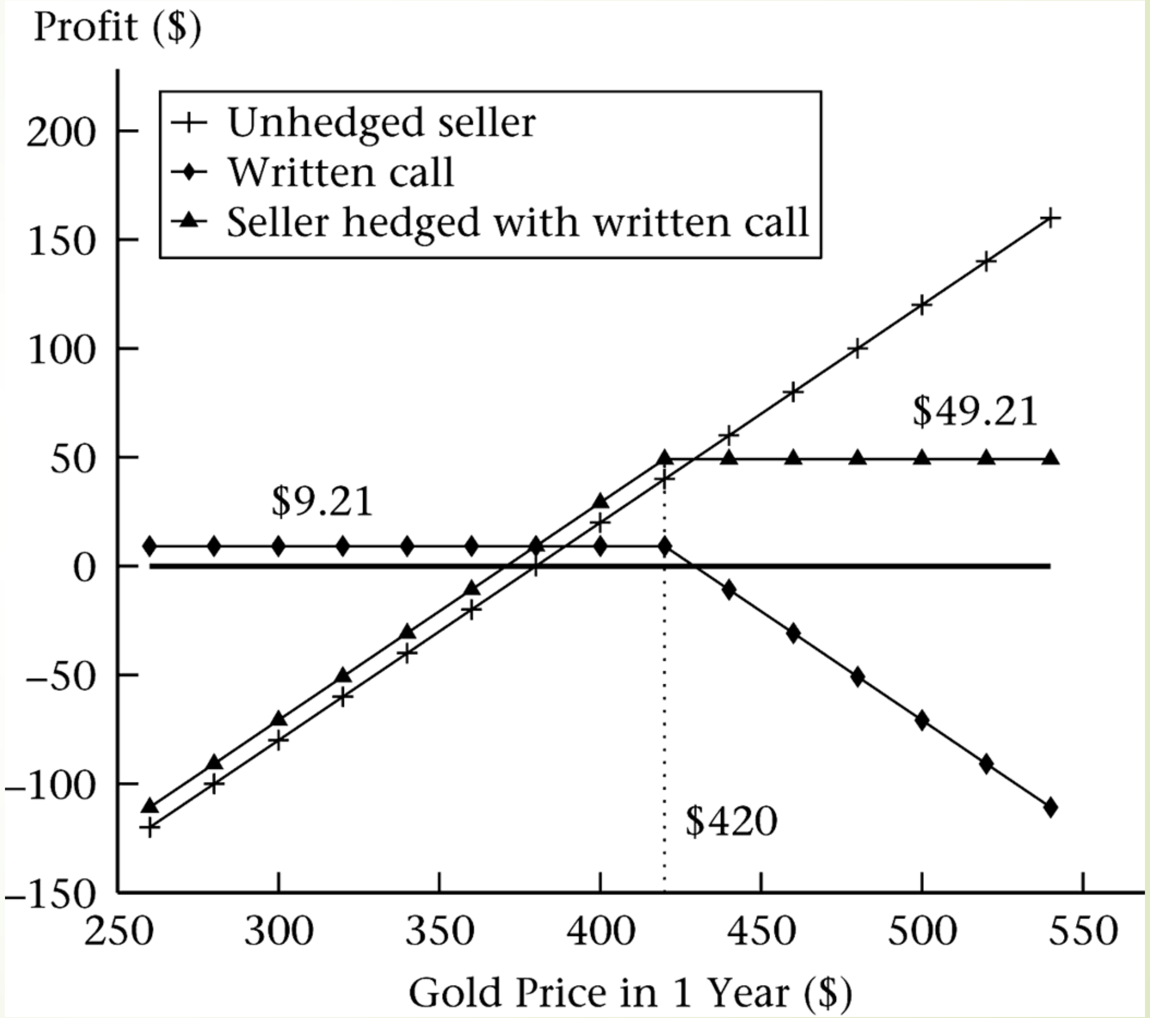
A gold-mining firm purchases a 420-strike put at the premium of $8.77 interest rate is 5%. FV=9.21

Producer: Insuring by Selling a Call

A **written** call reduces losses through a premium, but limits possible profits by providing a cap on the price.

<u>Example</u>:

A gold-mining firm sells a 420-strike call and receives an $8.77 premium (FV=$9.21)



The Buyer's Perspective

- A buyer that faces price risk on an input has an inherent short position in this commodity.

- When the price of, say raw material, is up the firm's profitability falls.

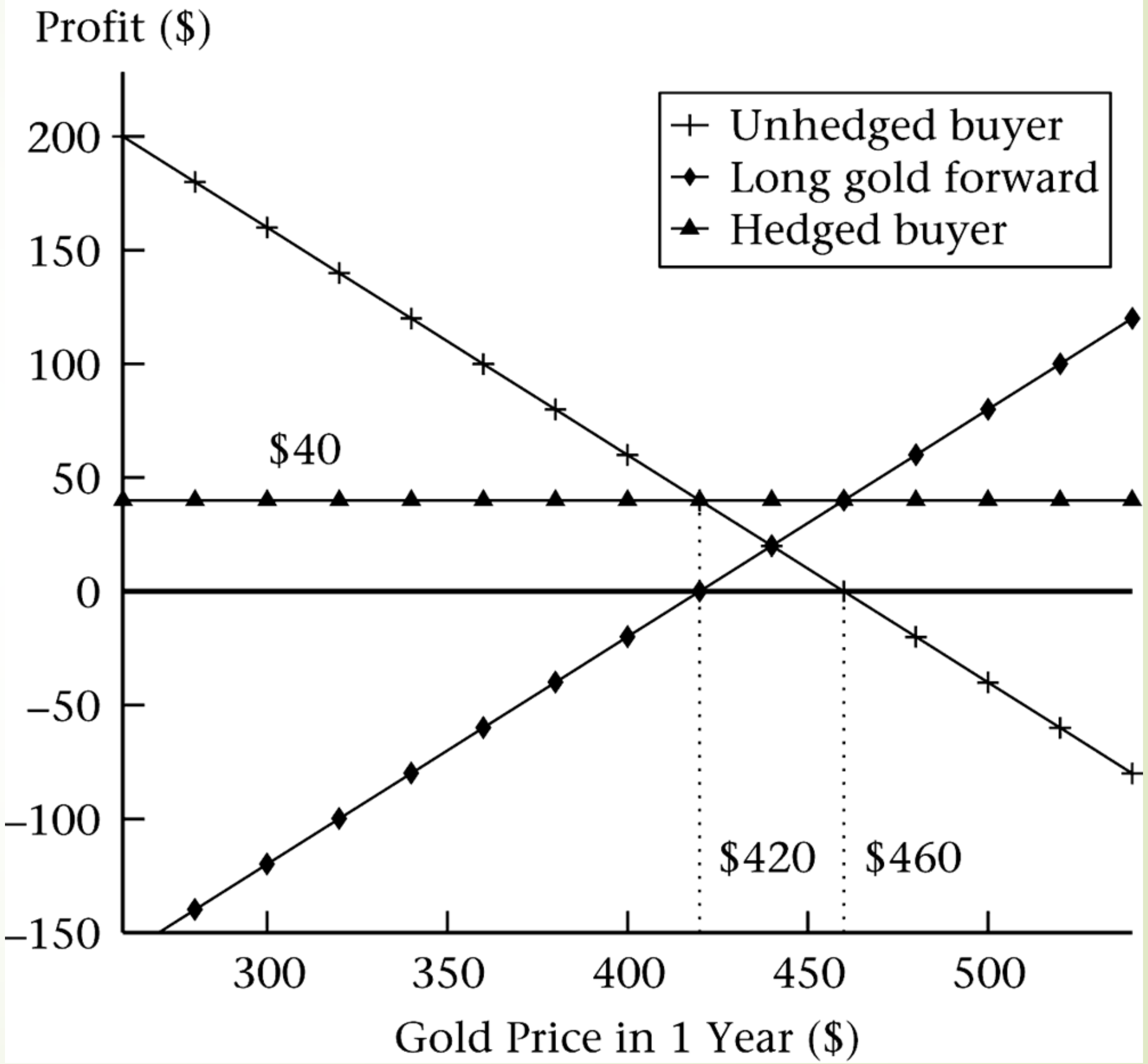
- Some strategies to hedge profit:
  - Buying forward.
  - Buying calls.

Buyer: Hedging With a Forward Contract

A **long forward contract** allows a buyer to lock in a price for his input.

<u>Example</u>:

A firm, using gold as an input, purchases a forward contract, agreeing to buy gold at a price of \$420/oz. in 1 year. The product is selling for \$460

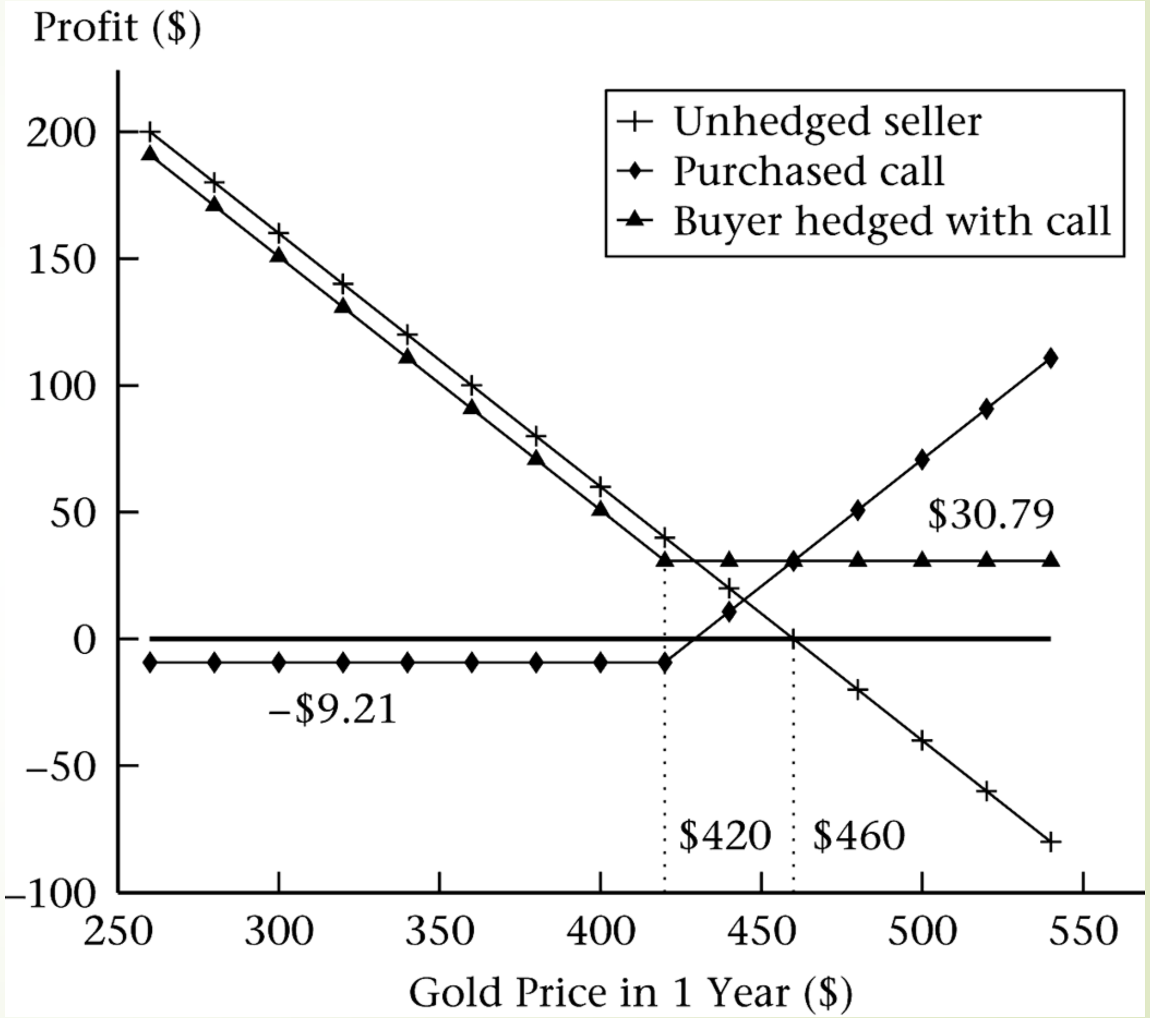


Buyer: Hedging With a Call Option

Buying a call option allows a buyer to have higher profits at low input prices, while being protected against high prices.

Example:

A firm, which uses gold as an input, purchases a 420-strike call at the premium (future value) of $9.21/oz.



Starbucks: Short Hedging with Futures Contracts

- Suppose Starbucks has an inventory of about 950,000 pounds of coffee, valued at $0.57 per pound.

- Starbucks fears that the price of coffee will fall in the short run, and wants to protect the value of its inventory.

Starbucks: Short Hedging with Futures Contracts

- How best to do this? You know the following:
  - There is a coffee futures contract at the New York Board of Trade.
  - Each contract is for 37,500 pounds of coffee.
  - Coffee futures price with three month expiration is $0.58 per pound.
  - Selling futures contracts provides current inventory price protection.
- 25 futures contracts covers **937,500** pounds.
- 26 futures contracts covers **975,000** pounds.

…

Starbucks: Short Hedging with Futures Contracts

…

- Starbucks decides to sell 25 near-term futures contracts.
- Over the next month, the price of coffee falls. Starbucks sells its inventory for $0.51 per pound.
- The futures price also falls, to $0.52. (There are two months left in the futures contract)
- How did this short hedge perform?
- That is, how much protection did selling futures contracts provide to Starbucks?

The Short Hedge Performance

Date	Starbucks Coffee Inventory Price	Starbucks Inventory Value	Near-Term Coffee Futures Price	Value of 25 Coffee Futures Contracts
Now	$0.57	$541,500	$0.58	$543,750
1-Month From now	$0.51	$484,500	$0.52	$487,500
Gain (Loss)	($0.06)	($57,000)	$0.06	$56,250

- The hedge was not perfect.
- But, the short hedge "threw-off" cash ($56,250) when Starbucks needed some cash to offset the decline in the value of their inventory ($57,000).

What would have happened if prices had increased by $0.06 instead?

Empirical Evidence on Hedging

- Half of nonfinancial firms report using derivatives.
- Among firms that do use derivatives, less than 25% of perceived risk is hedged, with firms likelier to hedge short-term risk.
- Firms with more investment opportunities are more likely to hedge.
- Firms that use derivatives have a higher market value and more leverage.

Option versus Futures Contracts

Options differ from futures in major ways:

- Holders of call options have no obligation to buy the underlying asset. Holders of put options have no obligation to sell the underlying asset.
- To avoid this obligation, buyers of calls and puts must pay a price today.
- Holders of futures contracts do not pay for the contract today, but they are obligated to buy or sell the underlying asset, depending upon the position.

Option Relations and Option Pricing

We will show the important put call parity then study two ways for pricing options:

- The binomial Tree.
- The B&S Formula.

Put-Call Parity

If the underlying asset is a stock and Div is the dividend stream, then

$$C(K,T) = P(K,T) + \left[S_0 - PV_{0,T}(Div)\right] - e^{-rT}(K)$$

Put Call Parity: An arbitrage Opportunity

Stock Price = 110

Call Price = 17

Put Price = 5

Effective (not cc) six-month interest rate 5%.

Maturity = .5 yr

K = 105

$$C - P > S_0 - K / (1 + r_f)^T$$

$$17 - 5 > 110 - (105 / 1.05)$$

$$12 > 10$$

Since the leveraged equity is less expensive, acquire the low cost alternative and sell the high cost alternative.

Put-Call Parity Arbitrage (assuming no dividends)

Position	Immediate Cashflow	Cashflow in Six Months $S_T<105$	$S_T \geq 105$
Buy Stock	-110	S_T	S_T
Borrow $K/(1+r)^T = 100$	+100	-105	-105
Sell Call	+17	0	$-(S_T - 105)$
Buy Put	-5	$105 - S_T$	0
Total	2	0	0

Parity for Options on Stocks (cont'd)

Examples:

- Price of a non-dividend paying stock: \$40, r=8%, option strike price: \$40, time to expiration: 3 months, European call: \$2.78, European put: \$1.99.

$$\$2.78 = \$1.99 + \$40 - \$40e^{-0.08\times0.25}$$

- Additionally, if the stock pays \$5 just before expiration, call: \$0.74, and put: \$4.85.

$$\$0.74 = \$4.85 + (\$40 - \$5e^{-0.08\times0.25}) - \$40e^{-0.08\times0.25}$$

Parity for Options on Stocks (cont'd)

Synthetic security creation using parity

- Synthetic stock: buy call, sell put, lend PV of strike and dividends
- Synthetic T-bill: buy stock, sell call, buy put (interesting tax issues)
- Synthetic call: buy stock, buy put, borrow PV of strike and dividends
- Synthetic put: sell stock, buy call, lend PV of strike and dividends

The Relation between Future, Strike, Call, and Put prices

- Suppose we have call and put options on the same underlying asset, which does not pay dividends, with the same time expiration and with the same strike price.
- The put call parity implies that

$$C - P = S - PV(K)$$

- Suppose now we have a future contract on the same underlying asset and with the same time to expiration. The futures price is F. What is the relation between F and K?

The Relation between Future, Strike, Call, and Put prices

To answer this question let us consider the following strategy.

You buy call, sell put, and sell the futures contract.

	$S_T < K$	$S_T \geq K$
Buy Call	0	$S_T - K$
Sell Put	$S_T - K$	0
Sell Futures	$F - S_T$	$F - S_T$
Total	$F - K$	$F - K$

The Relation between Future, Strike, Call, and Put prices

- The bottom line is that you pay C-P today and you get F-K in the future. Thus

$$C - P = PV(F - K)$$

- This is actually a generalization of the put call parity.

The Generalized Put-Call Parity

For European options with the same strike price and time to expiration the parity relationship is

Call – Put = *PV* (forward price – strike price)

Or

$$C(K,T) - P(K,T) = PV_{0,T}\left(F_{0,T} - K\right) = e^{-rT}\left(F_{0,T} - K\right)$$

Example

- Consider buying the 6-month 1000 strike call for a premium of \$93.808 and selling a similar put for \$74.201. What must the future price (set up today) be if the 6-month interest rate is 2%?

$$93.808 - 74.201 = PV(F - 1000). \text{ Thus } F = 1020.$$

- Are our previous hedging examples consistent with the relation we derived?

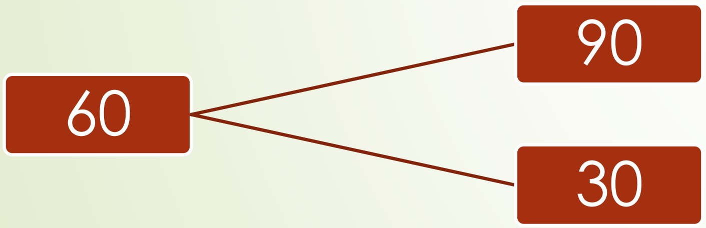
Binomial Option Pricing

- So far we have discussed how the price of one option is related to the price of another, but we have not addressed the question: how is the price of an option related to the price of the underlying asset.
- The binomial tree, a very general approach to valuing options, explicitly addresses this question.
- We assume that in a given period of time, the price of the underlying asset (e.g., a stock) will either increase to a particular value or decrease to another value.
- We will illustrate this approach with an example, similar yet different from the text example.

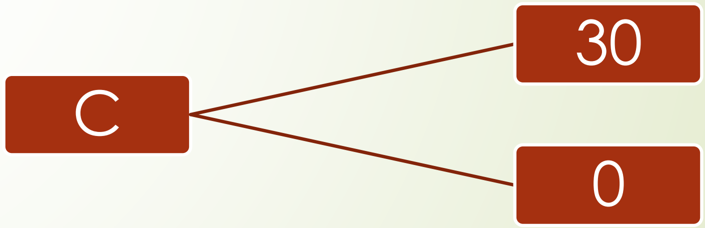
Binomial Option Pricing Example

Suppose that we have a stock with a price of \$60 today and that in one year, the price will be either \$90 or \$30. The risk-free rate is 10% (simple annualized rate). Consider the payoffs to a call option with a strike price of \$60.

Stock Price



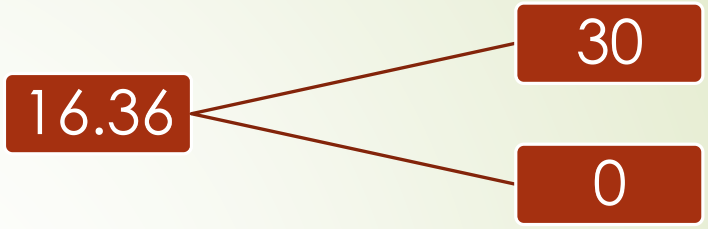
Call Option with K = 60



A Replicating Portfolio

Alternative Portfolio

- Buy .5 shares of stock for $30
- Borrow $13.64 (at 10% Rate)
- Net outlay $16.36



Payoff to this strategy:

	S=30	S=90
Value of .5 shares	15	45
Repay loan	-15	-15
Net Payoff	**0**	**30**

Payoffs are exactly the same as the Call. Therefore the call price should be $16.36. If not, then arbitrage is possible!

Another View of Replication of Payoffs and Option Values

- This example also shows us how options can be used to hedge the underlying asset.
- What if we held one share of stock and wrote 2 calls ($K = 60$)?

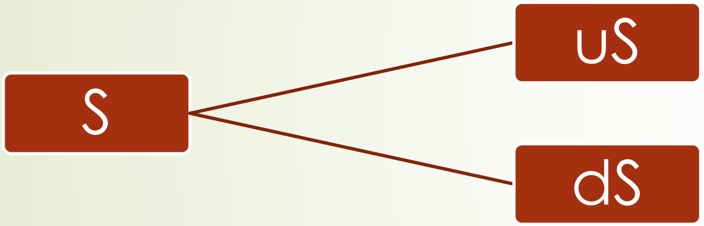
- Portfolio is perfectly hedged

Stock Value	30	90
Two Written Calls	0	-60
Net Payoff	**30**	**30**

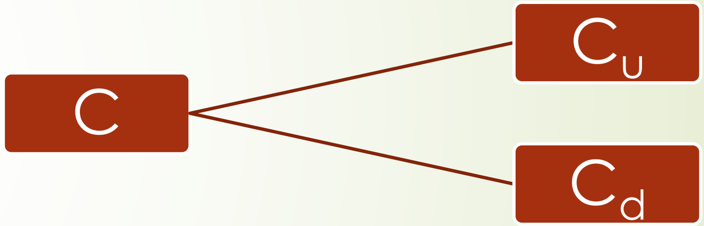
- The combined portfolio has a riskless return (note that this portfolio would cost ($60 - 2 \times 16.36 = 27.28$), so the return is 10%, i.e. the riskless rate of return.

Binomial Option Pricing: General One-period Case

Stock Price



Call Option with strike price K

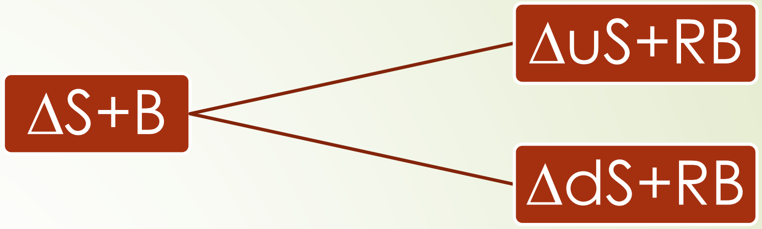


A General Replicating Portfolio

Alternative Portfolio

- Buy Δ shares of stock for \$ ΔS
- Put \$B into bonds (at r% Rate) (if B is negative, then borrow)
- Net outlay \$ $\Delta S + B$

Payoff to this strategy:



	S=uS	S=dS
Value of Δ shares	ΔuS	ΔdS
Bond (R=1+r)	RB	RB
Net Payoff	**$\Delta uS+RB$**	**$\Delta dS+RB$**

Solving for Δ and B

We want the replicating portfolio to gives us the same payoff as the option in both states:

$$\Delta uS + RB = C_u$$

$$\Delta dS + RB = C_d$$

$$\Delta = \frac{C_u - C_d}{uS - dS}$$

$$\mathrm{B} = \frac{uC_d - dC_u}{(u-d)R}$$

The One-Period Binomial Option Value Formula

The value of the call option today must be equivalent to the value of the replicating portfolio to prevent arbitrage.

$$C = \Delta S + B$$

$$= \frac{C_u - C_d}{u - d} + \frac{uC_d - dC_u}{(u - d)R}$$

$$= \frac{pC_u + (1 - p)C_d}{R}, \qquad where\ p = \frac{R - d}{u - d}$$

Risk-adjusted probabilities

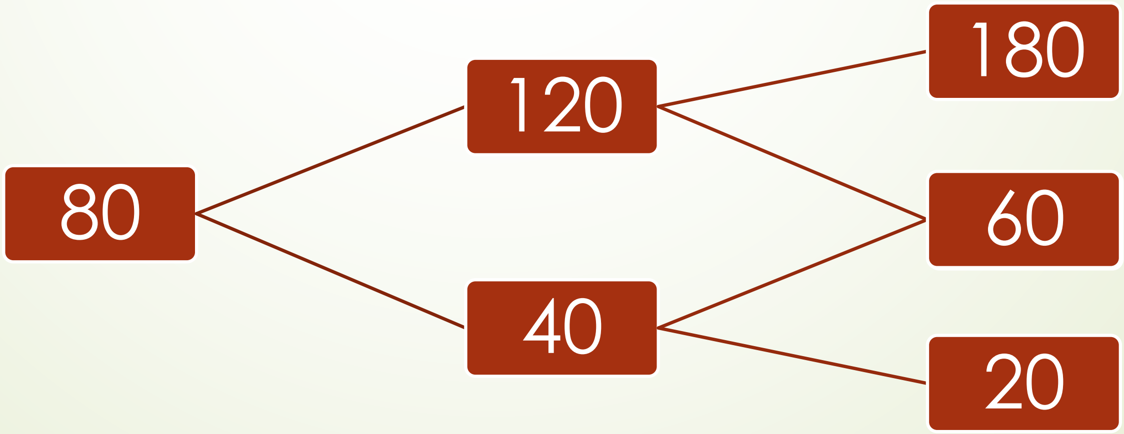
- Note that the valuation formula for the call is just a probability weighted average of the payoffs discounted by the risk-free rate. The adjustment for risk is taken into account in the "probabilities" that are used: p and (1-p).

- These probabilities can also be backed out by considering the underlying asset:

$$S = \frac{puS + (1-p)dS}{R} \quad \Rightarrow \quad p = \frac{R-d}{u-d}$$

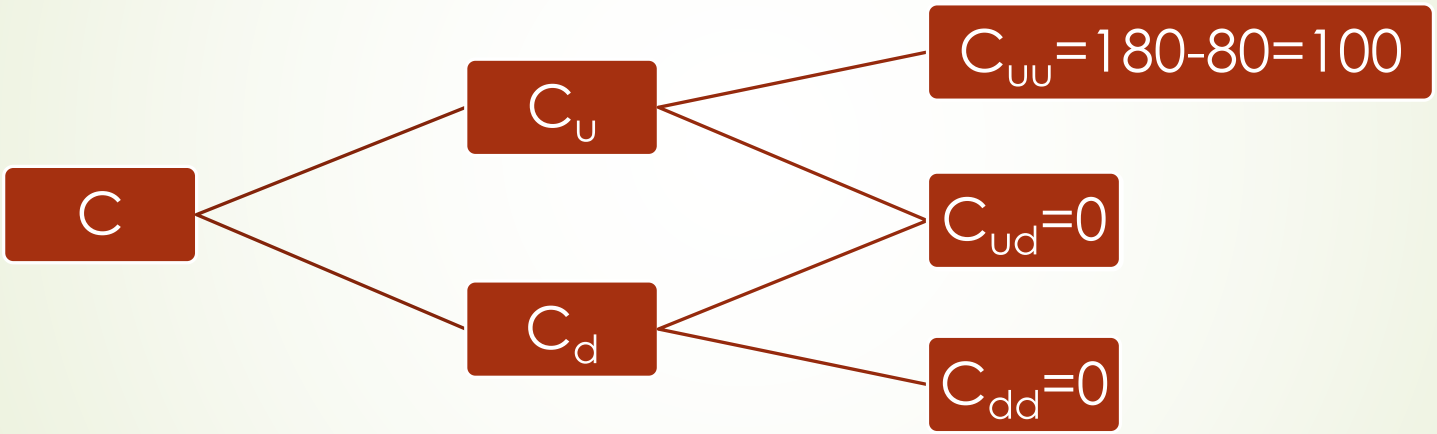
Generalizing the Binomial Approach

- While it may seem unrealistic that the price of the stock can only go up or down in a given period, we can break the life of the option into as many (very short) periods as we like!
- This point can be illustrated by using a two-period example (but can be generalized to as many periods as deemed necessary).
- Consider a stock that is selling today for 80, and which will go up or down by 50% in each of the next two periods (each of which is one-year long). The risk-free rate is 10% p.a. (i.e. R=1.10):



The binomial tree for the call option

Consider a call option with a strike price of $80.



Valuing C_u and C_d

- If we are in the up-state at the end of the first period, we are looking at a one-period tree.

- First, we need to calculate

$$p = \frac{R - d}{u - d} = \frac{1.10 - 0.5}{1.5 - 0.5} = 0.6$$

- Using our one-period formula:
  - $C_u = \frac{(0.6)\times 100 + (0.4)\times 0}{1.10} = 54.54$
  - $C_d = \frac{(0.6)\times 0 + (0.4)\times 0}{1.10} = 0$

Valuing C

- Now, consider the first period on its own. We now know the values of the options at the end of this first period (C_u and C_d).

- Using the same one-period valuation technique:

$$C = \frac{(0.6) \times (54.54) + (0.4) \times (0)}{1.10} = 29.75$$

The replicating portfolios

- Recall that we have formulas for Δ and B. We can use these at each point in the tree, based on the call and stock values in the up and down state at the end of each period.

- For example, at the beginning of the first period, we can calculate:

$$\Delta = \frac{C_u - C_d}{uS - dS} = \frac{54.54 - 0}{120 - 40} = 0.682$$

$$B = \frac{uC_d - dC_u}{(u-d)R} = \frac{0 - (0.5)(54.54)}{(1.5 - 0.5)1.1} = -24.793$$

Option Prices and Deltas

The tree below shows the prices and deltas (in parentheses) at each node.

Self Financing Strategy

- If the stock price is up you buy more stocks.
- You pay $(0.833\text{-}0.682)\times 120=18.2$
- You also pay interest on your loan $0.1\times 24.793=2.5$
- Who brings the money?
- Note that the new B = -45.45
- So the amount of debt increases by 20.7

Self Financing Strategy

- What if the stock price drops to 40.
- Then you sell the stock for $0.682 \times 40 = 27.3$
- You pay interest 2.5
- You net inflow is 24.8
- You also repay the loan 24.8
- Thus your net inflow is zero.

Put Options

- The analysis we have just gone through works exactly the same way for put options, or for any derivative security that depends on the stock price.

- Interpret C_u and C_d as the payoffs to a derivative security such as a call or put.

- In replicating a put, delta will be negative and B will be positive - i.e. we would short stock and lend out \$B.

- Note that $P = \Delta S + B$, or $B = P - \Delta S$, so the amount lent out is equal to the money from the short position plus the cost of the put option.

American Options

- The binomial valuation approach can also be used for valuing American Options (in fact, it must be used since formulas such as the Black-Scholes don't allow for early exercise).

- The only difference in the procedure is that at every point in the tree we need to check to see whether it is preferable to exercise the option early.

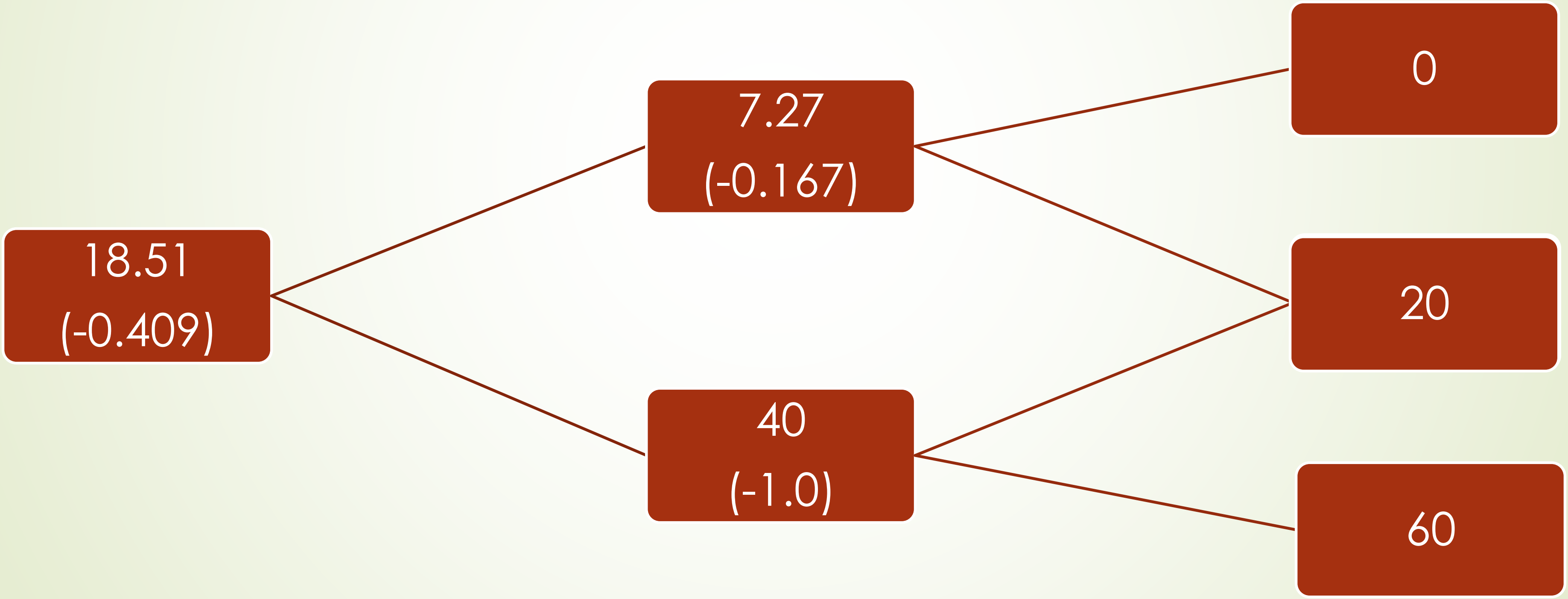
- In the following example, we value both a European and an American put option on the stock we just looked at, where the strike price equals $80.

European Put Option

Find the payoffs at maturity, and work backwards to find option and delta values.

American Put Option

In this case, check at each node to see whether it would be preferable to exercise (it is when the stock price goes down to $40 in the first year). Note that the American option is worth 2.64 more than the European option.

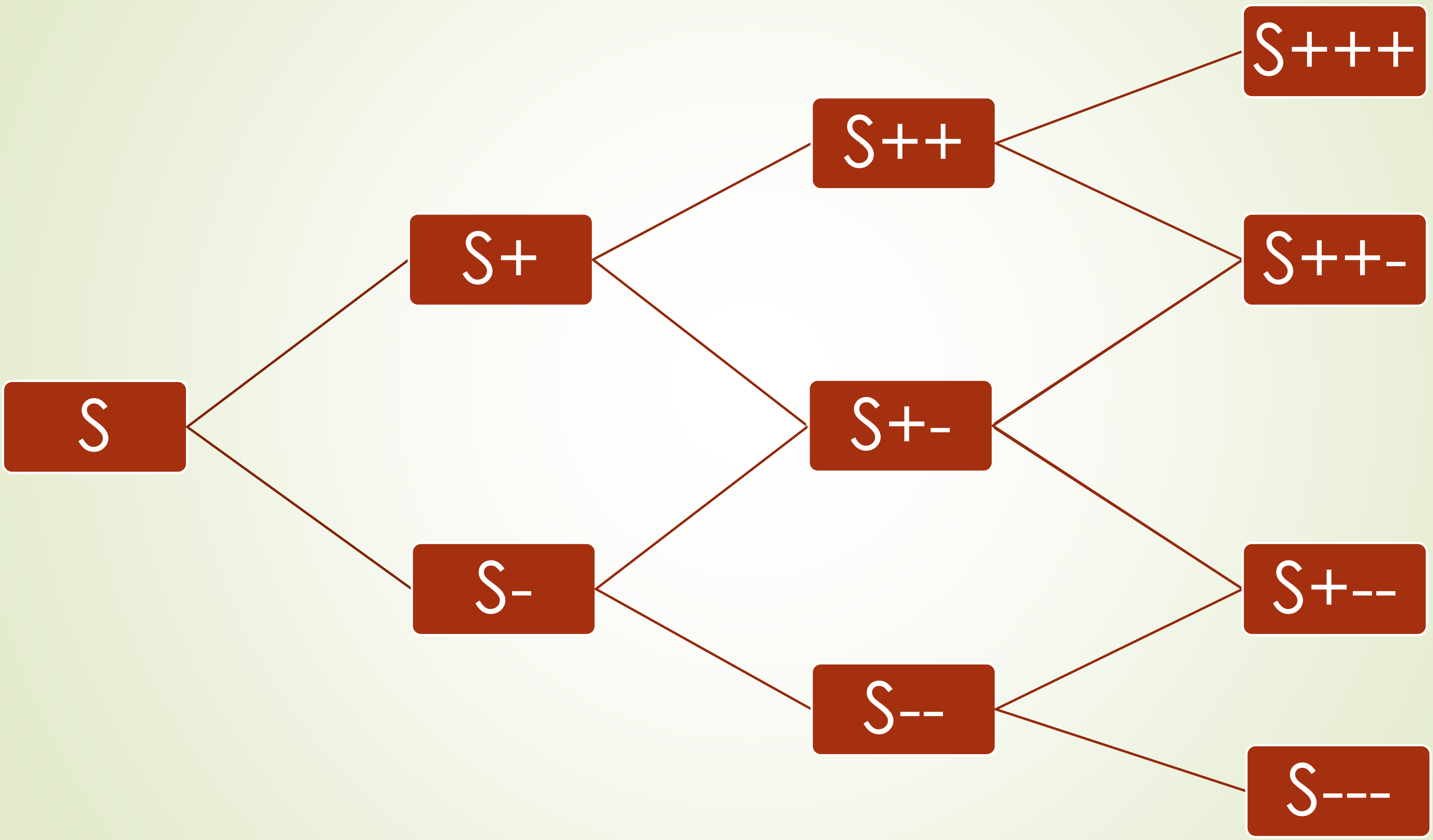


Exercising Options "Early"

Why may options be exercised early?

- As we just saw, puts may be exercised early if they are deep in-the-money. Since it is likely that the option will remain in-the-money, the holder would prefer to get the fixed strike price today rather than later in time.
- For call options, since the exercise price is paid out, there is an advantage, rather than disadvantage, to waiting in this regard (one would prefer to delay payment of a fixed amount).
- However, if (and only if) the stock pays out a dividend, the option holder may want to exercise early in anticipation of the stock price dropping at the ex-dividend date. However, the holder will only do so if the stock is deep enough in-the-money (or the option is near maturity) since he is giving up the time value of the option.

Expanding the number of periods



The Binomial Model - summary

- The binomial model can break down the time to expiration into potentially a very large number of time intervals, or steps.
- A tree of stock prices is initially produced working forward from the present to expiration.
- At each step it is assumed that the stock price will move up or down by an amount calculated using volatility and time to expiration. This produces a binomial distribution of underlying stock prices.
- The tree represents all the possible paths that the stock price could take during the life of the option.
- At the end of the tree, at expiration of the option, all the terminal option prices for each of the final possible stock prices are known as they simply equal their intrinsic values.

The Binomial Model - summary

- Next the option prices at each step of the tree are calculated working back from expiration to the present.
- The option prices at each step are used to derive the option prices at the next step of the tree using risk neutral valuation based on the probabilities of the stock prices moving up or down, the risk free rate and the time interval of each step.
- At the top of the tree you are left with one option price.

To get a feel …

- To get a feel of how the binomial tree work you can use the on-line binomial tree calculators:

- http://www.hoadley.net/options/binomialtree.aspx?tree=B

Nice features

- The big advantage of the binomial model is that it can be used to accurately price American options.
- This is because it's possible to check at every point in an option's life (at every step of the binomial tree) for the possibility of early exercise
- Where an early exercise point is found it is assumed that the option holder would elect to exercise, and the option price can be adjusted to equal the intrinsic value at that point. This then flows into the calculations higher up the tree and so on.
- The on-line binomial tree graphical option calculator highlights those points in the tree structure where early exercise would have caused an American price to differ from a European price.
- The binomial model basically solves the same equation, using a computational procedure that the Black-Scholes model solves using an analytic approach and in doing so provides opportunities along the way to check for early exercise for American options.

The Black-Scholes Model

- What happens if the number of periods in the binomial tree are greatly increased, and the periods become extremely short?
- If the volatility and interest rate in each period is the same, then we arrive at the model developed by Fischer Black and Myron Scholes.
- Call Option price

$$C(S,K,\sigma,r,T,\delta) = Se^{-\delta T}N(d_1) - Ke^{-rT}N(d_2)$$

- Put Option price

$$P(S,K,\sigma,r,T,\delta) = Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1)$$

- Where $d_1 = \frac{\ln(S/K)+\left(r-\delta+\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$

The Put Call Parity Revisits

Note that the put price easily follows by the implementing the put call parity and using the relations:

$$N(-d_1) = 1 - N(d_1)$$

$$N(-d_2) = 1 - N(d_2)$$

Option Pricing Parameters

The formula requires six parameters:

- Stock price, dividend, and volatility (stock level).
- Strike price and time to expiration (option level).
- Risky free rate (economy level).

Black-Scholes (BS) Assumptions

- Assumptions about stock return distribution:
  - Continuously compounded returns on the stock are normally distributed and independent over time (no "jumps").
  - The volatility of continuously compounded returns is known and constant.
  - Future dividends are known, either as dollar amount or as a fixed dividend yield.
- Assumptions about the economic environment:
  - The risk-free rate is known and constant.
  - There are no transaction costs or taxes.
  - It is possible to short-sell costlessly and to borrow at the risk-free rate.

Implied Volatility

- Volatility is unobservable.
- Choosing a volatility to use in pricing an option is difficult but important.
- One approach to obtaining a volatility is to use history of returns.
- However history is not a reliable guide to the future.
- Alternatively, we can invert the Black-Scholes formula to obtain option implied volatility.
- IV is the volatility implied by the option price observed in the market.
- Implied volatilities are not constant across strike prices and over time, in contrast to the B&S assumptions.

Volatility Index –VIX

- It provides investors with market estimates of expected volatility.
- It is computed by using near-term S&P 100 index options.

VIX Options

- A type of non-equity option that uses the VIX as the underlying asset.
- This is the first exchange-traded option that gives individual investors the ability to trade market volatility.
- A trader who believes that market volatility will increase can purchase VIX call options.
- Sharp increases in volatility generally coincide with a falling market, so this type of option can be used as a natural hedge.

Implied Volatility (cont'd)

…

- In practice implied volatilities of in, at, and out-of-the money options are generally different resulting in the volatility skew.
- Implied volatilities of puts and calls with same strike and time to expiration must be the same if options are European as implied by put-call parity.

Implied Volatility (cont'd)

Implied volatilities for S&P 500 options, 10/28/2004. Option prices (ask) from www.cboe.com; assume S=$1127.44, δ=1.85%, r=2%

Strike ($)	Expiration	Call Price ($)	Implied Volatility	Put Price ($)	Implied Volatility
1100	20/11/2004	34.30	0.1630	6.80	0.1575
1125	20/11/2004	17.10	0.1434	14.70	0.1447
1150	20/11/2004	5.80	0.1284	29.20	0.1339
1100	18/12/2004	41.70	0.1559	13.80	0.1539
1125	18/12/2004	24.50	0.1396	22.50	0.1436
1150	18/12/2004	13.00	0.1336	35.50	0.1351
1100	22/01/2005	49.10	0.1567	20.40	0.1513
1125	22/01/2005	33.00	0.1463	29.40	0.1427
1150	22/01/2005	20.00	0.1363	41.50	0.1337

Pricing Exotic Options

- An **exotic option** is a derivate which has features making it more complex than the commonly traded call and put options (vanilla options); These products are usually traded over-the-counter (OTC), or are embedded in structured notes.
- One can use the B&S structure to price a large set of such options.
- Below we display the price of four exotic options while the starting point is the B&S formula $C=BS(S,K,\sigma,r,T,\delta)$.
- That is, we will use the BS formula but will change one or more of the six underlying parameters. More details are coming up!

Option Payoff: Type I

$$\text{Option payoff} = \max[S^2 - K, 0]$$

- That is, at the time of expiration it is not the price of the underlying stock that matters – rather it is the squared stock price.
- The option price is then given by:

$$Price = BS\left(S^2, K, 2\sigma, r, T, -(r + \sigma^2)\right)$$

Option Payoff: Type II

$$\text{Option payoff} = \max[S_1 S_2 - K, 0]$$

- That is, at the time of expiration it is not the price of the underlying stock that matters – rather it is the product of two stock prices.
- The option price is then given by:

$$Price = BS\left(S_1 S_2, K, \sigma^*, r, T, -(r + \rho\sigma_1\sigma_2)\right)$$

$$\sigma^* = \sqrt{\sigma_1^2\sigma_2^2 + 2\rho\sigma_1\sigma_2}$$

Option Payoff: Type III

$$\text{Option payoff} = \max\left[\frac{S1}{S2} - K, 0\right]$$

- That is, at the time of expiration it is not the price of the underlying stock that matters – rather it is the ratio of two stock prices.
- The option price is then given by:

$$Price = BS\left(S_1/S_2, K, \sigma^*, r, T, r - \sigma_2(\sigma_2 - \rho\sigma_1)\right)$$

$$\sigma^* = \sqrt{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

Option Payoff: Type IV

$$\text{Option payoff} = \max[\sqrt{(S_1 S_2)} - K, 0]$$

- That is, at the time of expiration it is not the price of the underlying stock that matters – rather it is the geometric average of two stock prices.

- The option price is then given by:

$$Price = BS\left(\sqrt{S_1 S_2}, K, \sigma^*, r, T, -\frac{1}{8}(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)\right)$$

$$\sigma^* = \frac{1}{2}\sqrt{\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2}$$

Shortfall Probability and Option Pricing

- We now use concepts coming from option pricing to understand the risk of return of long horizon investment decisions.
- Previously, we were dealing with the mean variance setup to make invest decisions.
- Here, we introduce a new concept: investing based on shortfall probability.
- Let us denote by R the return on the investment over several years (say T years).

Shortfall Probability in Long Horizon

- Let us denote by R the cumulative return on the investment over several years (say T years).

- Rather than finding the distribution of R we analyze the distribution of

$$r = \ln(1 + R)$$

which is the continuously compounded return over the investment horizon.

- The investment value after T years is

$$V_T = V_0(1 + R_1)(1 + R_2) \dots (1 + R_T)$$

Shortfall Probability in Long Horizon

- Dividing both sides of the equation by V_0 we get

$$\frac{V_T}{V_0} = (1+R_1)(1+R_2)\dots(1+R_T)$$

thus

$$1+R = (1+R_1)(1+R_2)\dots(1+R_T)$$

- Taking natural log from both sides we get

$$r = r_1 + r_2 + \cdots + r_T$$

- Using properties of the normal distribution, we get

$$r \sim N(T\mu, T\sigma^2)$$

Shortfall Probability and Long Horizon

- Let us now understand the concept of shortfall probability.
- We ask: what is the probability that the investment yields a return smaller than the riskfree rate, or any other threshold level?
- To answer this question we need to compute the value of a riskfree investment over the T year period.
- The value of such a riskfree investment is

$$V_{r_f} = V_0\left(1 + R_f\right)^T = V_0 \exp\left(T r_f\right)$$

- where r_f is the continuously compounded risk free rate.

Shortfall Probability and Long Horizon

- Essentially we ask: what is the probability that
$$V_T < V_{r_f}$$
- This is equivalent to asking what is the probability that
$$\frac{V_T}{V_0} < \frac{V_{r_f}}{V_0}$$
- This, in turn, is equivalent to asking what is the probability that
$$\ln\left(\frac{V_T}{V_0}\right) < \ln\left(\frac{V_{r_f}}{V_0}\right)$$

Shortfall Probability and Long Horizon

- So we need to work out

$$p\left(\mathrm{r} < T_{r_f}\right)$$

- Subtracting $T\mu$ and dividing by $\sqrt{T}\sigma$ both sides of the inequality we get

$$P\left(z < \sqrt{T}\left(\frac{r_f - \mu}{\sigma}\right)\right)$$

- We can denote this probability by

$$Shortfall\ probability\ y = N\left(\sqrt{T}\left(\frac{r_f - \mu}{\sigma}\right)\right)$$

- Typically $r_f < \mu$ which means the probability diminishes with increasing T.

Example

Take $r=0.04$, $\mu=0.08$, and $\sigma=0.2$ per year. What is the Shortfall Probability for investment horizons of 1, 2, 5, 10, and 20 years?

Use the Excel NORMDIST function.

- If T=1 SP=0.42
- If T=2 SP=0.39
- If T=5 SP=0.33
- If T=10 SP=0.26
- If T=20 SP=0.19

One more Example

A stock portfolio's monthly continuously compounded return has a mean of 0.01 and a volatility of 0.05. if the current portfolio value is \$50 million, what is the probability that the portfolio's value will be less than \$65 millions in five years?

$$
\begin{aligned}
prob\left\{V_t < \frac{65}{50}\right\} &= prob\left\{V_t < 1.3\right\} \\
&= prob\left\{\ln V_t < 0.262\right\} \\
&= prob\left\{z < \frac{0.262 - T\mu}{\sqrt{T\sigma^2}}\right\} \\
&= prob\left\{z < \frac{0.262 - 60(0.01)}{\sqrt{60}(0.05)}\right\} \\
&= prob\left\{z < -0.872\right\} \\
&= 0.192
\end{aligned}
$$

Cost of Insuring against Shortfall

- Let us now understand the mathematics of insuring against shortfall.
- Without loss of generality let us assume that

$$V_0 = 1$$

- The investment value at time T is a given by the random variable V_T
- Once we insure against shortfall the investment value after T years becomes
  - If $V_T > \exp(Tr_f)$ you get V_T
  - If $V_T < \exp(Tr_f)$ you get $\exp(Tr_f)$

Cost of Insuring against Shortfall

- So you essentially buy an insurance policy that pays 0 if $V_T > \exp(Tr_f)$
  Pays $\exp(Tr_f) - V_T$ if $V_T < \exp(Tr_f)$

- You ultimately need to price a contract with terminal payoff given by
$$\max\{0, \exp(Tr_f) - V_T\}$$

- This is a European put option expiring in T years with
  a. S=1
  b. $\text{K} = \exp(T \cdot r_f)$.
  c. Riskfree rate given by r_f.
  d. Volatility given by σ
  e. Dividend yield given by $\delta = 0$

Cost of Insuring against Shortfall

- From the B&S formula we know that

$$Put = K\exp(-Tr_f)N(-d_2) - S\,exp(-\delta T)N\,(-d_1)$$

- Which becomes

$$Put = N\left(\frac{1}{2}\sigma\sqrt{T}\right) - N\left(-\frac{1}{2}\sigma\sqrt{T}\right)$$

- The B&S option-pricing model gives the current put price P as

$$Put = N(d_1) - N(d_2)$$

where

$$d_1 = \frac{\sigma\sqrt{T}}{2}, \qquad d_2 = -d_1$$

and $N(d)$ is $prob\{z < d\}$

Cost of Insuring against Shortfall

- For $\sigma = 0.2$ (per year)

T (years)	P
1	0.08
5	0.18
10	0.25
20	0.35
30	0.42
50	0.52

- The cost of the insurance increases in T, even though the probability of needing it decreases in T (if $\mu > r$).

Open questions

- Do you believe in time diversification?
- What are the difficulties with using shortfall probability as a risk measure and tool for asset allocation?
- We finish up this option section with the so-called option Greeks.

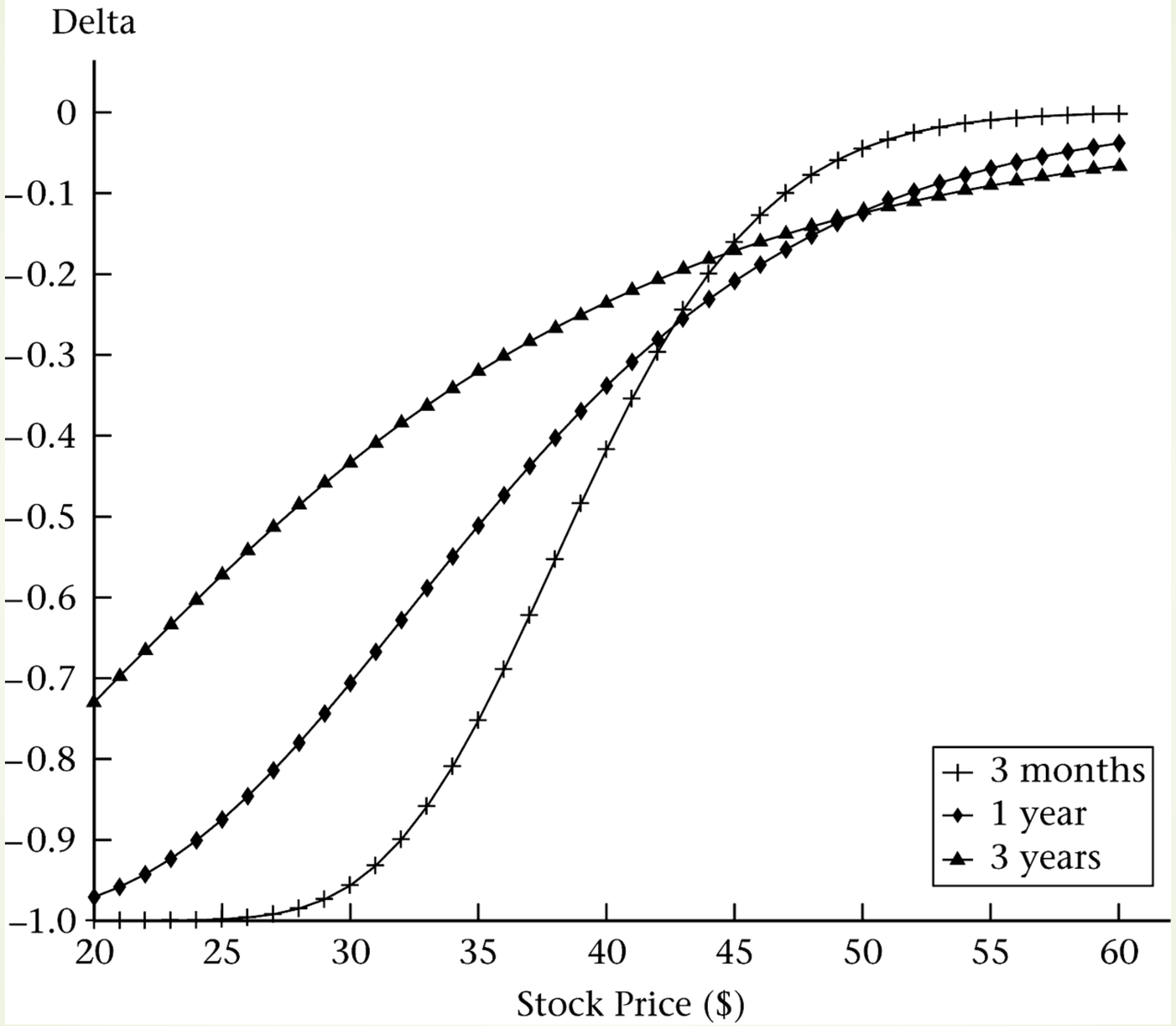
Option Greeks

- What happens to option price when *one* input changes?
  - Delta (Δ): change in option price when stock price increases by \$1.
  - Gamma (Γ): change in delta when the stock price increases by \$1.
  - $Vega$: change in option price when volatility increases by 1%.
  - Theta (Θ): change in option price when time to maturity decreases by 1 day.
  - Rho (ρ): change in option price when interest rate increases by 1%
- Greek measures for portfolios
  - The Greek measure of a portfolio is weighted average of Greeks of individual portfolio components. For example,

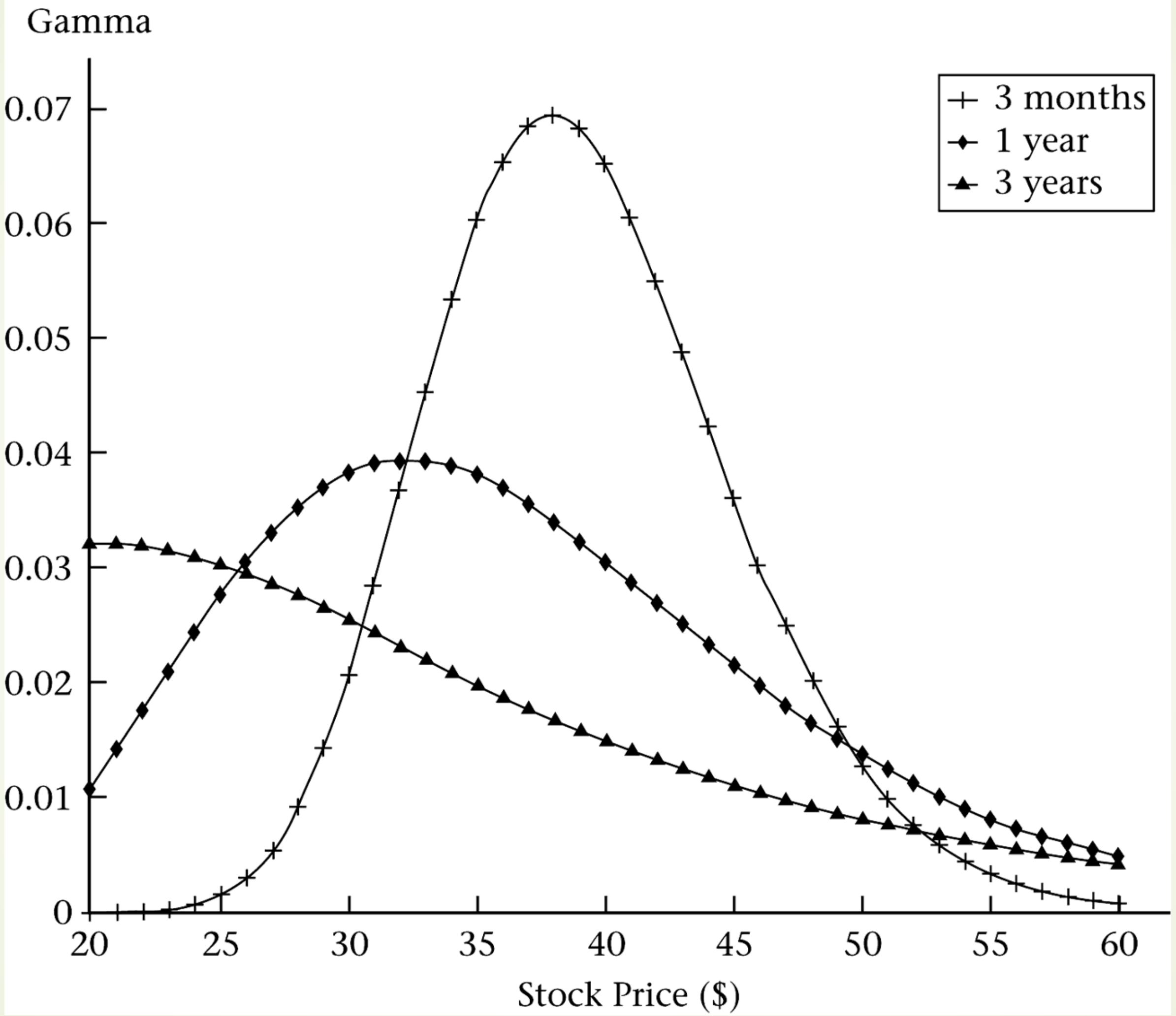
$$\Delta_{portfolio} = \sum_{i=1}^{n} \omega_i \Delta_i$$

- The following illustrations are based on strike price=\$40, sigma=30%, riskfree rate=8%, and no dividend payments.

Delta - Call

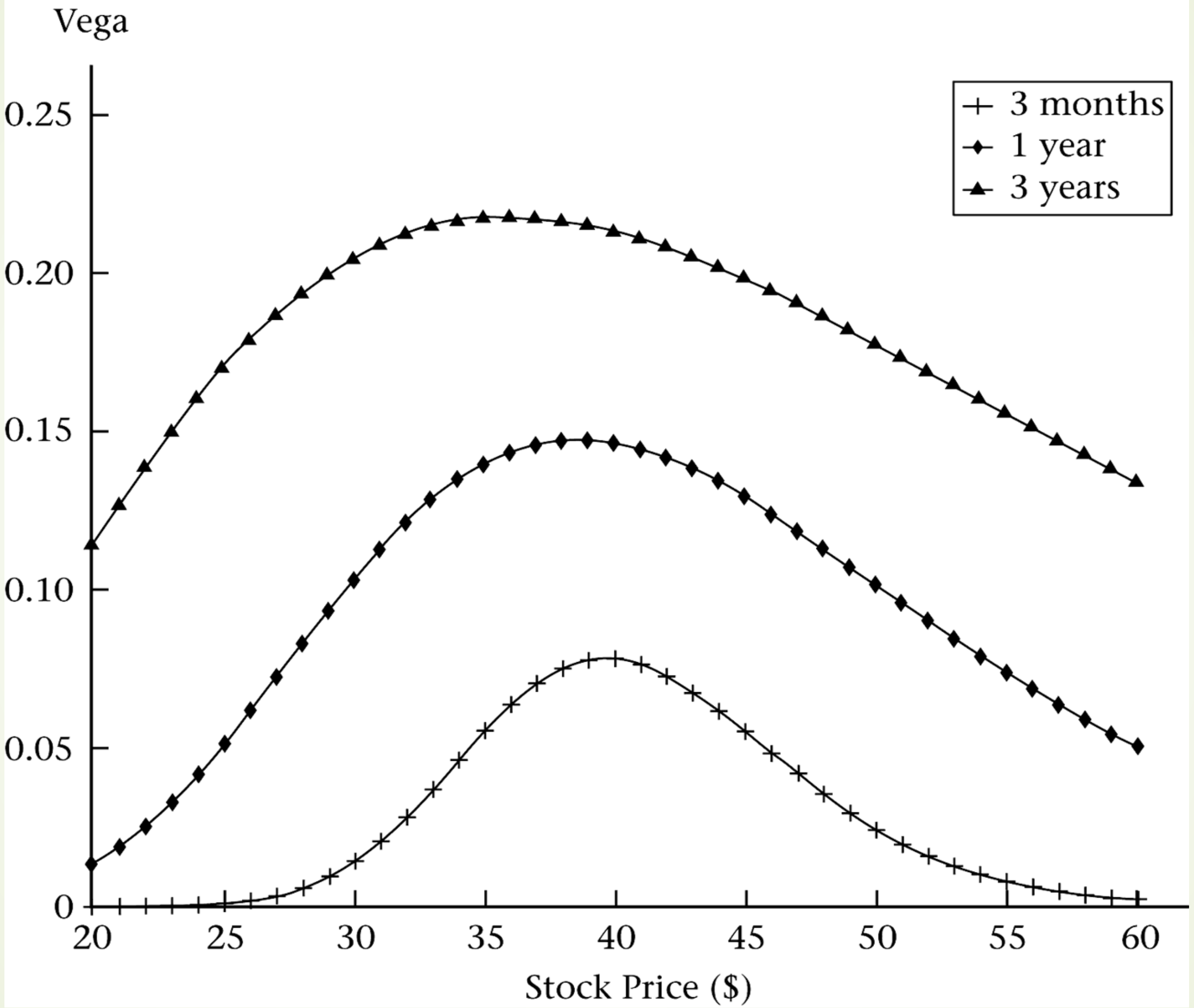

Delta
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
−1.0
20
25
30
35
40
45
50
55
60
Stock Price ($)
3 months
1 year
3 years

Gamma - Call



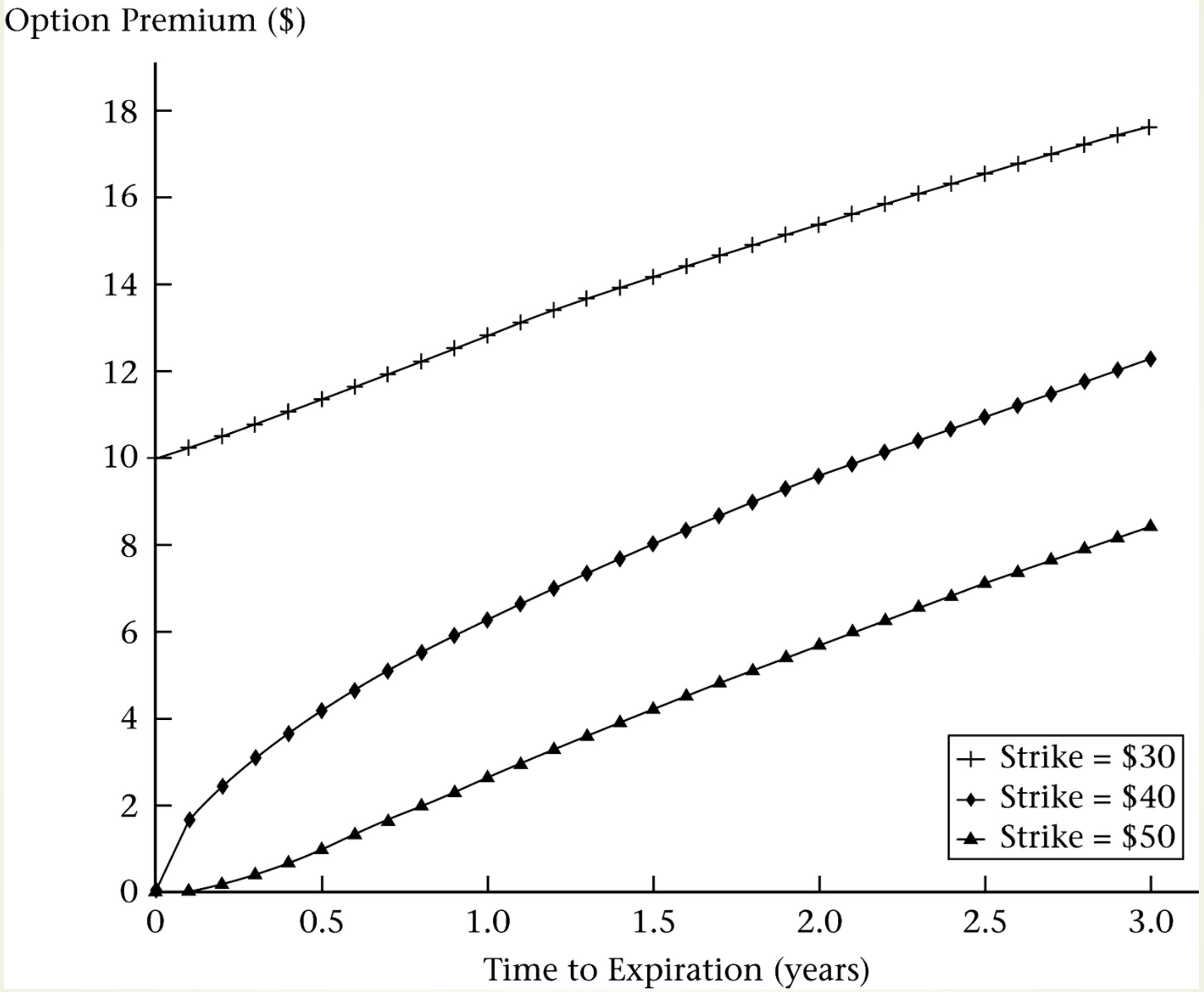
Gamma
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
20
25
30
35
40
45
50
55
60
Stock Price ($)
3 months
1 year
3 years

Option Greeks (cont'd)

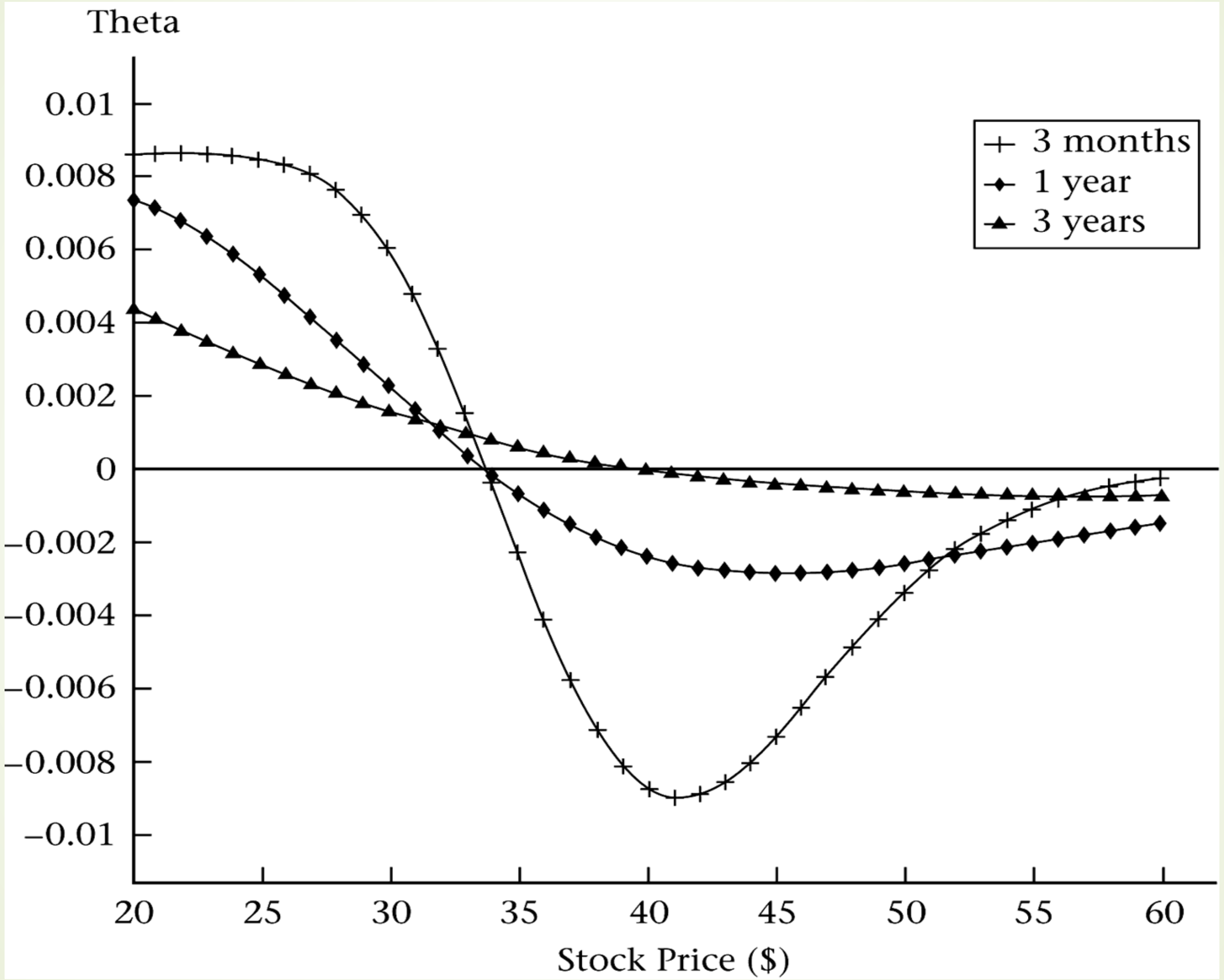


Option Greeks (cont'd)

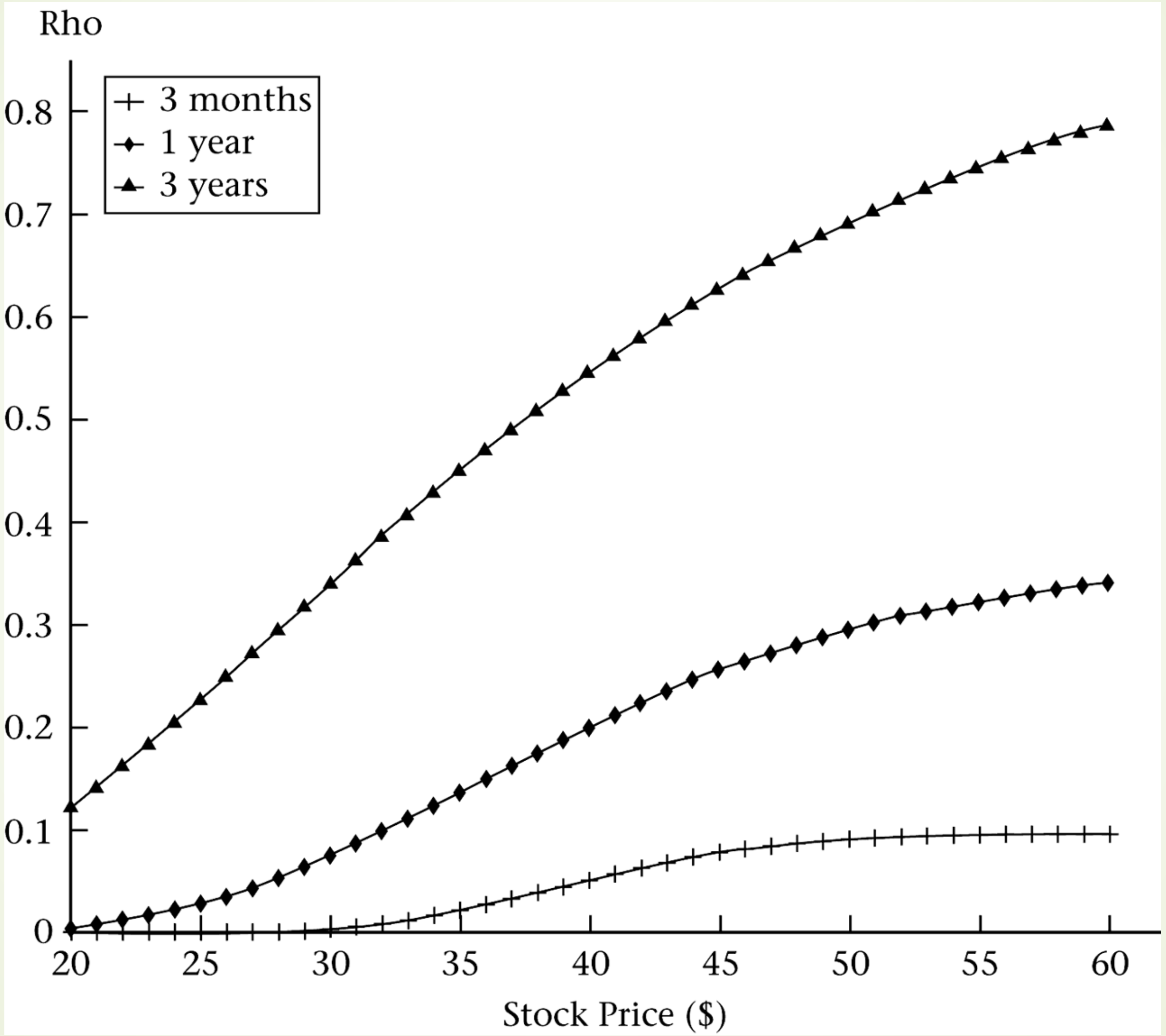
Stock price=$40



Option Greeks (cont'd)



Option Greeks (cont'd)



Option Greeks (cont'd)

Greeks for the bull spread examined in chapter 3 where

$$S = \$40, \sigma = 0.3, R = 0.08, T = 91\ days$$

with a purchased 40-strike call and a written 45-strike call. The column titled "combined" is a difference between column 1 and column 2.

	Option 1	Option 2	Combined
Wi	1	-1	-
Price	2.7804	0.9710	1.8094
Delta	0.5824	0.2815	0.3009
Gamma	0.0652	0.0563	0.0088
Vega	0.0780	0.0674	0.0106
Theta	-0.0173	-0.0134	-0.0040
Rho	0.0511	0.0257	0.0255

Option Greeks (cont'd)

Option elasticity (Ω):

- Ω describes the risk of the option relative to the risk of the stock in percentage terms: If stock price (S) changes by 1%, what is the percent change in the value of the option (C)?

$$\Omega \equiv \frac{\%\, change\ in\ C}{\%\, change\ in\ S} = \frac{\frac{change\ in\ C}{C}}{\frac{change\ in\ S}{S}} = \frac{\frac{change\ in\ S \times \Delta}{C}}{\frac{change\ in\ S}{S}} = \frac{S\Delta}{C}$$

Option Greeks (cont'd)

Example: S = \$41, K = \$40, σ = 0.30, r = 0.08, T = 1, d = 0

- Elasticity for call:

$$\Omega = S\Delta/C \;=\; \$41 \times 0.6911 \,/\, \$6.961 \;=\; 4.071$$

- Elasticity for put:

$$\Omega = S\Delta/C \;=\; \$41 \times (-\,0.3089 \,/\, \$2.886) \;=\; -\,4.389$$

Option Greeks (cont'd)

Option elasticity (Ω) (cont'd)

- The volatility of an option

$$\sigma_{option} = \sigma_{stock} \times \Omega$$

- The risk premium of an option

$$\gamma - r = (a - r) \times \Omega$$

- The Sharpe ratio of an option.

Option Greeks (cont'd)

Where | . | is the absolute value,

g: required return on option,

a: expected return on stock, and

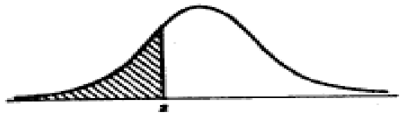
r: risk-free rate.

$$\text{Sharpe ratio for call } = \frac{\alpha - r}{\sigma} = \text{Sharpe ratio for stock.}$$

Appendix

TABLE I Values of the Standard Normal Distribution Function

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}u^2\right) du = P(Z \le z)$$



z	0	1	2	3	4	5	6	7	8	9
−3.	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.C102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0126	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0238	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0300	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0570	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
− .9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
− .8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
− .7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
− .6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
− .5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
− .4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
− .3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
− .2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
− .1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
− 0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Appendix

TABLE 1 Values of the Standard Normal Distribution Function (Continued)

z	0	1	2	3	4	5	6	7	8	9
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9430	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9648	.9656	.9664	.9671	.9678	.9686	.9693	.9700	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9762	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9874	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

Note 1: If a normal variable X is not "standard," its values must be "standardized": $Z = (X - \mu)/\sigma$, i.e., $P(X \leq x) = \Phi[(x - \mu)/\sigma]$.
Note 2: For "two-tail" probabilities, see Table Ib.
Note 3: For $z \geq 4$, $\Phi(z) = 1$ to four decimal places; for $z \leq -4$, $\Phi(z) = 0$ to four decimal places.
Note 4: Entries opposite 3 and -3 are for 3.0, 3.1, 3.2, etc., and -3.0, -3.1, etc., respectively.

End-of-Chapter Questions

To solve the next three questions, consider the three-period Binomial Tree framework when the stock price at time zero is 40, U=1.5, D=0.6, and R=1.1. Based upon these inputs answer the following four questions:

- **Q1**: What is the price of a European put option with a strike price of 100?
- **Q2**: What is the price of a European call option with a strike price of 100? (Hint: If you feel strongly about your answer for the put price you can implement the put-call parity.)
- **Q3**: What is the price of an American Put option with a strike price of 100?

Questions

- **Q4**: Assume that the prices of the underlying stock, call, and put options are 110, 17, and 8, respectively. In addition, the strike price is 105, the time to maturity is 0.5 years, and the annual risk free rate is 10.25%. Which of the following statements is correct?
  a. The call price must be too expensive
  b. The put price must be too expensive
  c. The put and call are properly priced
  d. The leveraged equity is correctly priced
  e. By buying the call and selling the put you make an arbitrage profit.

Questions

- **Q5**: In an economy there are three states of nature: fast growth, normal growth, and recession. The annual risk free rate in that economy is 5%. You buy a portfolio of three binary options. The first option pays $100 if the economy is in recession but zero otherwise. The second option pays $100 if the economy is in a fast growth state but zero otherwise. The third option pays $100 if the economy is in a normal growth state but zero otherwise. How much did you pay to but these three binary options?

Equity Valuation

Common Stock Valuation

- Our goal in this chapter is to examine the methods commonly used by financial analysts to assess the economic value of common stocks.

- These methods are grouped into two categories:
  - Dividend discount models.
  - Price ratio models.

Security Analysis: Be Careful Out There

- **Fundamental analysis** is a term for studying a company's accounting statements and other financial and economic information to estimate the economic value of a company's stock.

- The basic idea is to identify "undervalued" stocks to buy and "overvalued" stocks to sell.

- In practice, however, such stocks may in fact be correctly priced for reasons not immediately apparent to the analyst.

The Dividend Discount Model

The **Dividend Discount Model** (**DDM**) is a method to estimate the value of a share of stock by discounting all expected future dividend payments. The basic DDM equation is:

$$V(0) = \frac{D(1)}{(1+k)} + \frac{D(2)}{(1+k)^2} + \frac{D(3)}{(1+k)^3} + \cdots + \frac{D(T)}{(1+k)^T}$$

In the DDM equation:

- $V(0)$ = the present value of all future dividends.
- $D(t)$ = the dividend to be paid t years from now.
- k = the appropriate risk-adjusted discount rate.

Example: The Dividend Discount Model

- Suppose that a stock will pay three annual dividends of \$200 per year, and the appropriate risk-adjusted discount rate, k, is 8%.

$$V(0) = \frac{D(1)}{(1+k)} + \frac{D(2)}{(1+k)^2} + \frac{D(3)}{(1+k)^3}$$

- In this case, what is the value of the stock today?

$$V(0) = \frac{\$200}{(1+0.08)} + \frac{\$200}{(1+0.08)^2} + \frac{\$200}{(1+0.08)^3} = \$515.42$$

The Dividend Discount Model: the Constant Growth Rate Model

- Assume that the dividends will grow at a constant growth rate g. The dividend next period (t + 1) is:

$$D(t+1) = D(t) \times (1+g)$$

So, $$D(2) = D(1) \times (1+g) = D(0) \times (1+g) \times (1+g)$$

- For constant dividend growth, the DDM formula becomes:

$$V(0) = \frac{D(0)(1+g)}{k-g}\left[1 - \left(\frac{1+g}{1+k}\right)^T\right] \qquad if\ k \neq g$$

$$V(0) = T \times D(0) \qquad if\ k = g$$

Example: The Constant Growth Rate Model

- Suppose the current dividend is \$10, the dividend growth rate is 10%, there will be 20 yearly dividends, and the appropriate discount rate is 8%.

$$V(0) = \frac{D(0)(1+g)}{k-g}\left[1 - \left(\frac{1+g}{1+k}\right)^T\right] \quad if\ k \neq g$$

- What is the value of the stock, based on the constant growth rate model?

$$V(0) = \frac{\$10 \times (1.10)}{0.08 - 0.10}\left[1 - \left(\frac{1.10}{1.08}\right)^{20}\right] = \$243.86$$

The Dividend Discount Model: the Constant Perpetual Growth Model

- Assuming that the dividends will grow **forever** at a constant growth rate g.

- For constant perpetual dividend growth, the DDM formula becomes:

$$V(0) = \frac{D(0)(1+g)}{k-g} = \frac{D(1)}{k-g} \qquad (important\colon g < k)$$

Example: Constant Perpetual Growth Model

- Think about the electric utility industry.
- In mid-2005, the dividend paid by the utility company, American Electric Power (AEP), was \$1.40.
- Using D(0)=\$1.40, k = 7.3%, and g = 1.5%, calculate an estimated value for DTE.

$$V(0) = \frac{\$1.40 \times (1.015)}{0.073 - 0.015} = \$24.50$$

Note: the actual mid-2005 stock price of AEP was \$38.80.

What are the possible explanations for the difference?

Example: Constant Perpetual Growth Model

- Think about the electric utility industry.
- In 2007, the dividend paid by the utility company, DTE Energy Co. (DTE), was $2.12.
- Using D(0) =$2.12, k = 6.7%, and g = 2%, calculate an estimated value for DTE.

$$P_0 = \frac{\$2.12 \times (1.02)}{0.067 - 0.02} = \$46.01$$

Note: the actual mid-2007 stock price of DTE was $47.81.

Quite close!

The Dividend Discount Model: Estimating the Growth Rate

The growth rate in dividends (g) can be estimated in a number of ways:

- Using the company's historical average growth rate.
- Using an industry median or average growth rate.
- Using the *sustainable growth rate*.

The Historical Average Growth Rate

Suppose the Kiwi Company paid the following dividends:

Year	Dividend
2000	$1.50
2001	$1.70
2002	$1.75
2003	$1.80
2004	$2.00
2005	$2.20

The spreadsheet below shows how to estimate historical average growth rates, using arithmetic and geometric averages.

2005	$2.20	10.00%		
2004	$2.00	11.11%		
2003	$1.80	2.86%	Grown at	
2002	$1.75	2.94%	Year:	7.96%:
2001	$1.70	13.33%	2000	$1.50
2000	$1.50		2001	$1.62
			2002	$1.75
	Arithmetic Average:	8.05%	2003	$1.89
			2004	$2.04
	Geometric Average:	7.95%	2005	$2.20

The Sustainable Growth Rate

$$\text{Sustainable Growth Rate} = \text{ROE} \times \text{Retention Ratio}$$
$$= \text{ROE} \times (1\text{-Payout Ratio})$$

- Return on Equity (ROE) = Net Income / Equity
- Payout Ratio = Proportion of earnings paid out as dividends.
- Retention Ratio = Proportion of earnings retained for investment.

Example: Calculating and Using the Sustainable Growth Rate

- In 2005, American Electric Power (AEP) had an ROE of 14.59%, projected earnings per share of \$2.94, and a per-share dividend of \$1.40. What was AEP's:
  - **Retention rate?**
  - **Sustainable growth rate?**
- Payout ratio = \$1.40 / \$2.94 = .476
- So, retention ratio = 1 – 0.476 = 0.524 or 52.4%
- Therefore, AEP's sustainable growth rate = 0.1459 × 52.4% = 7.645%

Example: Calculating and Using the Sustainable Growth Rate

- What is the value of AEP stock, using the perpetual growth model, and a discount rate of 7.3%?

$$V(0) = \frac{\$1.40 \times (1.07645)}{0.073 - 0.07645} = -\$436.82 \ll \$38.80$$

- **Recall the actual mid-2005 stock price of AEP was $38.80**
- Clearly, there is something wrong because we have a negative price.
- What causes this negative price?
- Suppose the discount rate is appropriate. What can we say about g?

Example: Calculating and Using the Sustainable Growth Rate

- In 2007, AEP had an ROE of 10.17%, projected earnings per share of \$2.25, and a per-share dividend of \$1.56. What was AEP's:
  - **Retention rate?**
  - **Sustainable growth rate?**
- Payout ratio = \$1.56 / \$2.25 = 0.693
- So, retention ratio = 1 – 0.693 = 0.307 or 30.7%
- Therefore, AEP's sustainable growth rate=$.1017 \times .307 = .03122$, or 3.122%

Example: Calculating and Using the Sustainable Growth Rate, Cont.

- What is the value of AEP stock, using the perpetual growth model, and a discount rate of 6.7%?

$$P_0 = \frac{\$1.56 \times (1.03122)}{0.067 - 0.03122} = \$44.96$$

- **The actual mid-2007 stock price of AEP was $45.41.**
- In this case, using the sustainable growth rate to value the stock gives a reasonably accurate estimate.
- What can we say about g and k in this example?

The Two-Stage Dividend Growth Model

- The two-stage dividend growth model assumes that a firm will initially grow at a rate g_1 for T years, and thereafter grow at a rate $g_2 < k$ during a perpetual second stage of growth.
- The Two-Stage Dividend Growth Model formula is:

$$V(0) = \frac{D(0)(1+g_1)}{k-g_1}\left[1-\left(\frac{1+g_1}{1+k}\right)^T\right] + \left(\frac{1+g_1}{1+k}\right)^T \frac{D(0)(1+g_2)}{k-g_2}$$

Using the Two-Stage Dividend Growth Model, I.

- Although the formula looks complicated, think of it as two parts:
  - Part 1 is the present value of the first T dividends (it is the same formula we used for the constant growth model).
  - Part 2 is the present value of all subsequent dividends.

- So, suppose MissMolly.com has a current dividend of D(0) = \$5, which is expected to “shrink” at the rate g_1 =-10% for 5 years, but grow at the rate g_2 = 4% forever.

- With a discount rate of k = 10%, what is the present value of the stock?

Using the Two-Stage Dividend Growth Model, II.

$$V(0) = \frac{D(0)(1+g_1)}{k-g_1}\left[1-\left(\frac{1+g_1}{1+k}\right)^T\right] + \left(\frac{1+g_1}{1+k}\right)^T \frac{D(0)(1+g_2)}{k-g_2}$$

$$V(0) = \frac{\$5.00(0.90)}{0.10-(-0.10)}\left[1-\left(\frac{0.9}{1+0.10}\right)^5\right] + \left(\frac{0.90}{1+0.10}\right)^5 \frac{\$5.00(1+0.04)}{0.10-0.04}$$

=\$14.25+\$31.78

=\$46.03

The total value of \$46.03 is the sum of a \$14.25 present value of the first five dividends, plus a \$31.78 present value of all subsequent dividends.

Example: Using the DDM to Value a Firm Experiencing "Supernormal" Growth, I.

- Chain Reaction, Inc., has been growing at a phenomenal rate of 30% per year.
- You believe that this rate will last for only three more years.
- Then, you think the rate will drop to 10% per year.
- Total dividends just paid were $5 million.
- The required rate of return is 20%.
- What is the total value of Chain Reaction, Inc.?

Example: Using the DDM to Value a Firm Experiencing "Supernormal" Growth, II.

- First, calculate the total dividends over the "supernormal" growth period:

Year	Total Dividend: (in $millions)
1	$5.00 x 1.30 = $6.50
2	$6.50 x 1.30 = $8.45
3	$8.45 x 1.30 = $10.985

- Using the long run growth rate, g, the value of all the shares at Time 3 can be calculated as:

$$V(3) = [D(3) \times (1 + g)] / (k - g)$$

$$V(3) = [\$10.985 \times 1.10] / (0.20 - 0.10) = \$120.835$$

Example: Using the DDM to Value a Firm Experiencing "Supernormal" Growth, III.

Therefore, to determine the present value of the firm today, we need the present value of $120.835 and the present value of the dividends paid in the first 3 years:

$$V(0) = \frac{D(1)}{(1+k)} + \frac{D(2)}{(1+k)^2} + \frac{D(3)}{(1+k)^3} + \frac{V(3)}{(1+k)^3}$$

$$V(0) = \frac{\$6.50}{(1+0.20)} + \frac{\$8.45}{(1+0.20)^2} + \frac{\$10.985}{(1+0.20)^3} + \frac{\$120.835}{(1+0.20)^3}$$

=$5.42+$5.87+$6.36+$69.93

=$87.58 million.

Discount Rates for Dividend Discount Models

- The discount rate for a stock can be estimated using the capital asset pricing model (CAPM).
- We can estimate the discount rate for a stock using this formula:

Discount rate = time value of money + risk premium

= U.S. T-bill rate + (stock beta × stock market risk premium)

T-bill rate	return on 90-day U.S. T-bills
Stock Beta	risk relative to an average stock
Stock Market Risk Premium:	risk premium for an average stock

Observations on Dividend Discount Models, I.

Constant Perpetual Growth Model:

- Simple to compute
- Not usable for firms that do not pay dividends
- Not usable when $g > k$
- Is sensitive to the choice of g and k
- k and g may be difficult to estimate accurately.
- Constant perpetual growth is often an unrealistic assumption.

Observations on Dividend Discount Models, II.

Two-Stage Dividend Growth Model:

- More realistic in that it accounts for two stages of growth
- Usable when $g > k$ in the first stage
- Not usable for firms that do not pay dividends
- Is sensitive to the choice of g and k
- k and g may be difficult to estimate accurately.

Price Ratio Analysis, I.

- Price-earnings ratio (P/E ratio)
  - Current stock price divided by annual earnings per share (EPS).

- Earnings yield
  - Inverse of the P/E ratio: earnings divided by price (E/P).

- High-P/E stocks are often referred to as growth stocks, while low-P/E stocks are often referred to as value stocks.

Price Ratio Analysis, II.

- Price-cash flow ratio (P/CF ratio)
  - Current stock price divided by current cash flow per share.
  - In this context, cash flow is usually taken to be net income plus depreciation.

- Most analysts agree that in examining a company's financial performance, cash flow can be more informative than net income.

- Earnings and cash flows that are far from each other may be a signal of poor quality earnings.

Price Ratio Analysis, III.

- Price-sales ratio (P/S ratio)
  - Current stock price divided by annual sales per share.
  - A high P/S ratio suggests high sales growth, while a low P/S ratio suggests sluggish sales growth.

- Price-book ratio (P/B ratio)
  - Market value of a company's common stock divided by its book (accounting) value of equity.
  - A ratio bigger than 1.0 indicates that the firm is creating value for its stockholders.

Price/Earnings Analysis, Intel Corp.

- Intel Corp (INTC) - Earnings (P/E) Analysis.
  - 5 year average P/E ratio 37.30.
  - Current EPS \$1.16.
  - EPS growth rate 17.5%.

- **Expected stock price** = historical P/E ratio × projected EPS.
  - **\$50.84** = 37.30 × (\$1.16 × 1.175)
  - **Mid-2005 stock price = \$26.50**

Price/Earnings Analysis, Intel Corp.

- Intel Corp (INTC) - Cash Flow (P/CF) Analysis.
  - 5 year average P/CF ratio 19.75.
  - Current CFPS $1.94.
  - CFPS growth rate 13.50%.

- **Expected stock price** = historical P/CF ratio × projected CFPS.
  - **$43.49** = 19.75 × ($1.94 × 1.135)
  - **Mid-2005 stock price = $26.50**

Price/Sales Analysis, Intel Corp.

- Intel Corp (INTC) - Sales (P/S) Analysis.
  - 5-year average P/S ratio 6.77.
  - Current SPS \$5.47.
  - SPS growth rate 10.50%.

- **Expected stock price** = historical P/S ratio × projected SPS
  - **\$40.92** = 6.77 × (\$5.47 × 1.105)
  - **Mid-2005 stock price = \$26.50**