Uncertain imitability: an analysis of interfirm differences in efficiency under competition

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Causal ambiguity inherent in the creation of productive processes is modeled by attaching an irreducible ex ante uncertainty to the level of firm efficiency that is achieved by sequential entrants. Without recourse to scale economies or market power, the model generates equilibria in which there are stable interfirm differences in profitability, an above-normal industry rate of return, and a lack of entry even when firms are atomistic price-takers. The free-entry equilibrium for rational noncollusive firms is characterized for atomistic firms and for firms of fixed size, and some analytic results are obtained for the more realistic case in which firms have an arbitrary cost function. Numerical results for the associations implied between concentration, industry profitability, fixed entry costs, and the dispersion of firm profitabilities are obtained for selected cases.

1. Introduction

■ It has often been noted that the considerable uncertainty connected with major commercial ventures and *de novo* entry will produce a dispersion in the results obtained by different firms even when initial endowments are equivalent. The conventional view is that competition and free entry will eliminate such differences, so their persistence may be taken to indicate the presence of market power or impeded entry. However, if the original uncertainty stems from a basic ambiguity concerning the nature of the causal connections between actions and results, the factors responsible for performance differentials will resist precise identification. Under such conditions the uncertainty attaching to entry and imitative attempts persists and complete homogeneity is unattainable. Thus, persistent differentials in profitability may be consistent with free entry and fully competitive behavior.

This article formalizes these arguments by introducing a concept, labeled *uncertain imitability*, that allows analytic treatment of causal ambiguity and that generates interfirm heterogeneity as one of an industry's free-entry equilibrium properties. Modeling interfirm differences in size and profitability as stemming from stochastic events is not novel, but the approaches used to date have depended upon *ad hoc* specifications of boundedly rational behavior and have been partial equilibrium models, excluding entry.[1] By contrast, the models presented in this article posit free entry and independent profit-maximizing

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[1] See, for example, Mancke (1974), the work of Nelson and Winter (1975, 1978), and Albin and Alcaly's (1979) simulation. Futia's (1980) model of Schumpeterian competition does include entry, but assumes away interfirm differences to focus more clearly on concentration.

behavior. Even when firms are atomistic price-takers, we find that uncertain imitability can lead to supernormal industry profits together with a lack of entry. Additionally, uncertain imitability provides a theoretical connection between the height of this apparent "entry barrier" and the stable dispersion of interfirm profit rates. Finally, while the standard view is that excess industry profits induce entry, this theory suggests that high profits, *ceteris paribus*, may well signal the presence of very successful and difficult to imitate competitors and thereby impede rational entry attempts.

The sections that follow develop the assumptions underlying uncertain imitability, discuss the general case of atomistic firms, and explore two specific applications. In the first, firms are highly stylized: to obtain mathematically simple and intuitive results firms are restricted to a common fixed size and uncertainty attaches to the level of costs. In the second, the cost function is arbitrary, and uncertainty attaches to a scale factor. Here sequential entry generates a renewal process in industry size, and we develop the conditions under which a unique entry-limiting size exists. Finally, numerical results for specific cases are obtained to explore the implied associations among the level of uncertainty, the fixed cost of entry, the dispersion of firm profitabilities, industry profitability, and concentration.

2. Theory and assumptions

■ A theory explaining the dispersion of firm efficiencies within an industry must address both the origins of interfirm differences and the mechanisms that impede their elimination through competition and entry. In equilibrium changes in wealth are unexpected, so it is necessary to model net differences in firm efficiency as appearing randomly. For example, Williamson (1975, Ch. 11) discusses the rise of dominant firms through luck or the unexpected default of competitors. Caves and Porter (1977) attribute the rise of a group structure within industries to random initial differences in scale, skill, and preferences.

In mathematical models the randomness is usually attached to the outcomes of purposeful investment or research programs. Mancke (1974), taking issue with market power interpretations of observed correlations between market share and profitability, constructed a simulation in which the rate of return on each firm's capital budget was a random variable.[2] In a similar vein, Nelson and Winter (1978) studied the structural attributes of simulated Schumpeterian competition. In that model each firm received, by chance, research draws which allowed it to replace its current level of efficiency with a draw from a changing distribution (the latent technology).

The simulations of Mancke and of Nelson and Winter focused on the generation of heterogeneity among existing firms over time; neither permitted entry.[3] However, our interest is centered on the connection between interfirm heterogeneity and the conditions of entry, thereby requiring a shift in attention to the uncertainty faced by potential entrants. In this regard it is useful to distinguish between uncertainty of production and the uncertainty involved in the creation, discovery, or "production" of a new production function. In this article we equate the creation of a new firm (*de novo* entry or entrepreneurship) with the "production" of a production function, thereby positing an irreducible uncertainty in this process.

To simplify the present analysis we assume that industry demand is fixed and known, technology is stable, and the product is homogeneous. Firms are price-takers and neutral

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[2] See Caves, Gale, and Porter (1977) for a critical analysis of this position as well as Mancke's (1977) reply.

[3] Like the Mancke model, Nelson and Winter's simulation incorporates an assumption of capital market imperfections. In particular, firm growth is assumed to be constrained by the financing available through retained earnings and matching bank borrowing. In our opinion, capital market failures are the least important source of differential rents, market power, or concentration.

with regard to risk, using expected value as the criterion of choice. Uncertain imitability is operationalized by making a parameter of the firm's cost function depend upon a realization from a probability distribution. Each prospective entrant knows the distribution, but can only discover its actual cost function by making a nonrecoverable entry investment; after entry the firm's cost function is fixed and known. Entry decisions are globally optimal, and entry ceases when the expected net discounted value of entry is less than zero.

The assumption of uncertainty in the creation of new cost functions explains the origin of efficiency differences. The fact that the same uncertainty applies to all imitative and entry attempts explains their persistence despite free entry and raises the possibility that entry will cease before industry profits are eliminated.[4] Nevertheless, in neoclassical theory efficiency differences are also eliminated by competition among incumbents; techniques are imitated and the prices of factors found to be especially effective are bid up. If efficiency differences persist, they are traceable to imperfections in the factor markets. In general, markets for factors will be imperfect under conditions of uniqueness, ambiguity, or enforceable property rights to special factors.[5]

Ambiguity as to what factors are responsible for superior (or inferior) performance acts as a powerful block on both imitation and factor mobility. Demsetz (1973, p. 2) notes: "It is not easy to ascertain just why G.M. and I.B.M. perform better than their competitors. The complexity of these firms defies easy analysis, so that the inputs responsible for their success may be undervalued by the market for some time."[6] It might be argued that these inputs are undervalued because competitors fail to recognize them, which implies that the issue is just one of information sharing. Instead, we hold that it may never be possible to produce a finite unambiguous list of the factors of production responsible for the success of such firms. This ambiguity is not just a private embarrassment to economists, but is the heart of the matter. Factors of production cannot become mobile unless they are known.

The more common explanation of factor immobility is not ambiguity, but uniqueness combined with enforceable rights to the exclusive use of the unique resource (e.g., a patent on an invention, the ownership of a rich mineral deposit). Our model applies to such cases, but it is worth noting that the concepts of uncertainty and functional uniqueness (as opposed to purely nominal distinctiveness) are deeply interdependent; in the absence of uncertainty, the creation of a unique resource could be repeated and its uniqueness destroyed. This rich connection between uniqueness and ambiguity is also emphasized in Williamson's (1979) treatment of idiosyncratic investment. Frequent transactions between people or between people and complex tools give rise, he argues, to unique transaction-specific skills that are, to use Polanyi's word, unspecifiable (1958, p. 53). Here again we find factors of production that are immobile not only because they are unique, but also because their replication is a difficult and uncertain endeavor.

In our models the firms' inability to fully control the nature of their production functions introduces an element of "natural selection" into the long-term (free-entry) equilibrium: resources flow toward the most efficient firms, and the least efficient may be forced out of business. This sorting process, in which the "unfit" are swept away, is close to that envisioned by Alchian (1950). Our equilibrium, however, is equivalent neither

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[4] The idea that entry continues until industry profits disappear is a heuristic appropriate only under perfect neoclassical conditions. Eaton and Lipsey (1978) make this point strongly, and trace it back to Kaldor (1938). In addition, they document the continuing inappropriate use of the zero-rent condition when there are fixed costs or other indivisibilities.

[5] It is also true that transaction costs contribute to factor immobility, but Williamson (1979) has shown that high transaction costs are themselves outcomes of the underlying ambiguity and uniqueness.

[6] See also the comments on this subject by McGee (1974) and Schwartzman (1973). Both emphasize the importance of factor market imperfections in explaining profitability differences.

to that obtained under full neoclassical conditions[7] nor to the evolutionary equilibria of Nelson and Winter (1974, 1975, 1978, 1980) in which boundedly rational firms grope toward their goals in the face of changing conditions. In our model all firms act to maximize net expected wealth and some are just "luckier" than others.[8]

In summary, uncertain imitability obtains when the creation of new production functions is inherently uncertain and when either causal ambiguity or property rights in unique resources impede imitation and factor mobility. The concept does not apply when differences in efficiency are readily diminished through imitation or factor mobility (e.g., nonpatentable inventions, copyable product ideas). Although we have chosen to deal with cost functions and a stable environment in this article, the concept also applies to differentiated products and changing environments, albeit with substantial analytical difficulties. Finally, while the concept is most concrete at the level of the individual project, we believe that its relevance increases when very complex products and administrative structures are considered. Indeed, management is far from an exact science, and the ambiguity surrounding the linkage between action and performance in large firms virtually guarantees the existence of substantial uncertain imitability.

3. The atomistic case

■ The case of atomistic firms allows the simplest explication of our model of competitive entry under uncertain imitability. Let each firm's total costs be given by $T(q, b)$, where q is the rate of output and b, the carrier of uncertain imitability, is the firm's realization of the random variable X, where X has distribution function F. Without loss of generality, we assume that larger values of b are more "desirable." More specifically, for each given market-clearing price firm profits increase in b. We further assume that realizations of X are independent;[9] each prospective entrant into the industry faces the same uncertainty as to the value of b it will obtain. This uncertainty is resolved for each firm once and for all at the time of entry: some firms receive "poor" cost functions and other "lucky" firms find themselves in the possession of especially efficient facilities. Associated with each entry attempt (and thereby each realization of X) is a fixed nonrecoverable cost K, where $K > 0$. This represents expenses tied directly to the entry process and also includes the firm-specific (nonmarketable) investment required to create the firm. If a draw from F is sufficiently poor, the firm may choose (or be forced) to withdraw its entry attempt, forfeiting the amount K. Extant firms may reinvest in the industry by the same process as any other potential entrant: pay the entry cost K and draw from F to obtain a cost function.

We assume that there is an infinitely large group of identical possible entrants, their ordering in the queue of potential entrants being arbitrary. Entry is sequential; each potential entrant observes the results obtained by previous entrants and receives an independent draw from F.[10]

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[7] As Winter (1964) has pointed out, neoclassical and evolutionary equilibria will only coincide under special and restrictive conditions.

[8] One might, of course, choose to view uncertain imitability as the outcome of bounded rationality, but we prefer to distinguish between bounds on the quality of decisions and bounds on the quality of the best available theory on which decisions may be based. There is a difference, for example, between being unable to predict the exact size of an underground oil deposit, and being unable to work out the optimal drilling policy in the face of uncertainty.

[9] In assuming that realizations of X are independent and identically distributed, it would appear that we have ruled out learning. In reality, the assumption is relatively innocuous in this regard because X is defined as the *irreducible* uncertainty in postentry performance *after* learning has taken place.

[10] Because the firms are atomistic, the equilibrium necessitates an uncountable number of firms; in turn, this raises technical difficulties with regard to the independence assumption. To circumvent these difficulties—and retain all intuition—simply assume that $F(B)$ is the proportion of entering firms whose realization of b does not exceed B. From the viewpoint of each firm its realization of b has distribution F.

Associated with the industry is a known (strictly decreasing) demand function $D(p)$. If we imagine the industry as initially empty, it is intuitively clear that firms will enter, driving down the market price, until entry halts because the expected value of an additional entry attempt has become negative. At this final equilibrium, and at each point in the industry's expansion path, we assume that there is a competitive equilibrium determined by the demand curve and the supply function generated by the incumbent firms.

To characterize the instantaneous supply function, assume that once in the industry, firms act independently to maximize their profits. That is, observing a market price p, a firm with cost function $T(q, b)$ selects a level of output $q(p, b)$ which maximizes[11] $pq - T(q, b)$. Let $\pi(p, b) = pq(p, b) - T(q(p, b), b)$ be the stream of profits associated with continually confronting a price p with a cost function $T(q, b)$. Note that if a firm draws a value of b so low that its profits are negative, it cannot look forward to any improvements, since the market-clearing price can only remain constant or fall with additional entry. Letting $b_0(p)$ be the smallest solution to $\pi(p, b) = 0$, it is clear that the only firms present in an industry with price p have $b \geq b_0(p)$, the others having withdrawn from the industry, each suffering a loss K.

Given these assumptions, how may the optimal entry policy and the resultant industry equilibrium be characterized? Suppose that a potential entrant was certain to be the last entrant; together with atomism this would insure the constancy of the current price p. Letting r be the discount rate, $V(p)$, the expected net discounted value of entry is given by

$$V(p) = -K + (1/r) \int_{b_0(p)}^{\infty} \pi(p, b) dF(b). \tag{1}$$

The free-entry equilibrium occurs when the price level drops to a point which deters further entry. Since $V(p) < 0$ for some p and π is increasing in p, there will be a unique solution p^* to $V(p) = 0$. Entry ceases only when p falls below p^*, so that p^* is the free-entry equilibrium price. The optimal entry policy is thus quite simple in the atomistic case: firms attempt entry whenever the current price equals or exceeds p^*. Furthermore, there is a unique $b^* = b_0(p^*)$ such that a firm receiving $b < b^*$ will clearly be forced to withdraw by the time equilibrium is established. Here, and in the models that follow, we assume that entrants arrive like a "hail of bullets," the final equilibrium being achieved immediately. This simplification does not change the nature of the final equilibrium, but our analysis does ignore quasi rents earned by early entrants.[12]

Implicit in the above analysis is the assumption that the choke price exceeds p^*, that is, $D(p^*) > 0$. If not, then there will be no entry, and the industry will not exist. With the proviso $D(p^*) > 0$, it is clear from (1) that the industry's *long-run* supply curve is horizontal at p^*, and it does not depend upon D. A permanent shift in D, say to the right, would induce further entry while leaving the market-clearing price unchanged at p^*.

The probability that any given entry attempt succeeds is $1 - F(b^*)$. Because the attainment of a permanent position depends only upon receiving a cost function with b of at least b^*, the value of b for a firm in the mature industry is a random variable with distribution $F(x)/(1 - F(b^*))$ for $x \geq b^*$ and zero otherwise.

Under perfect competition each firm would receive profits at rate rK on its investment. The expected stream of rents (surplus profits) R_s received in equilibrium by a

[11] Although atomism suggests that $q(p, b)$ be the pure price-taking solution to the maximization problem, there is nothing in the model structure to prevent using Cournot, shared monopoly, or any other symmetric maximization concept.

[12] For a detailed discussion of the temporary rents obtained by early entrants see Lippman and Rumelt (1980).

surviving firm is

$$R_s = -rK + \frac{1}{1 - F(b^*)} \int_{b^*}^{\infty} \pi(p^*, b)dF(b), \tag{2}$$

which, when combined with (1), yields

$$R_s = \frac{F(b^*)}{1 - F(b^*)} rK. \tag{3}$$

Although the condition $V(p^*) = 0$ means that prospective entrants expect zero rents, the actual survivors of the entry process, those fortunate enough to draw $b \geq b^*$, will receive, on average, the level of rents given in (3). Although these firms are atomistic price-takers, entry does not drive the average survivor's rents to zero.

The survivor's rents R_s increase with increases in the failure probability $F(b^*)$; the greater the failure rate on entry, the greater will be the observed level of surplus profit in equilibrium.[13] However, because these firms are risk neutral, it would be incorrect to view R_s as a risk premium. The straightforward interpretation is that for each survivor there were $F(b^*)/(1 - F(b^*))$ unsuccessful entrants, each of which suffered a loss of K. Viewed in this light, R_s is simply the bias generated by only measuring the profits of the successful; before drawing from F, successful and unsuccessful entrants were indistinguishable.

If $\pi(p^*, b) \geq 0$ for all b, so that entrants never receive negative profits, $F(b^*) = 0$, and we obtain the classical result of zero expected rent among extant firms. The second moment of rent, however, does not vanish, and these atomistic firms will display a range of efficiencies in equilibrium.

Because $V(p)$ is increasing in p (recall that $\pi(p, b_0(p)) = 0$), we know that p^* increases with rK. Larger values of p^* imply smaller values of b^* and $F(b^*)$, the probability of failure. The connection between rK and R_s cannot be signed, but there is an unambiguous negative relationship between K and R_s/K, the survivor's excess rate of return. The observed rate of return on K decreases with K because an increased entry fee brings about an equilibrium with less failure; in turn, this lowers the relative premiums accorded to survivors.

At the center of our approach to modeling uncertain imitability is the *ex ante* uncertainty in postentry performance, and it is interesting to look at what happens as this uncertainty changes. Consider the distribution F_π of the random variable $\pi \equiv \pi(p, X)$, and rewrite (1) as

$$V(p) = -K + (1/r)E \max (0, \pi). \tag{1a}$$

The fact that $V(p)$ increases with mean-preserving increases in the risk of F_π easily follows from the definition of stochastic dominance.[14] Because $V(p)$ is also increasing in p, p^* falls with mean-preserving increases in the risk of F_π. *Increased uncertainty lowers the price and improves welfare in this model because it increases the chance that very efficient firms will appear.*

A slightly different interpretation of this result emerges by noticing that any change in the problem specification that reduces p^* would have private economic value were it available on a one-time basis to a single actor. Thus, a potential entrant would be willing

[13] Stonebraker (1976) measured a positive association between industry profitability and two measures of entry risk. However, he interpreted these as monopoly rents obtained through the protection of an entry barrier created by risk.

[14] If $\int_{-\infty}^{t} F_1(x)dx \leq \int_{-\infty}^{t} F_2(x)dx$ for all t, then F_2 is said to be riskier than F_1 (in the sense of second-order stochastic dominance). Moreover, if their means are equal, then F_2 represents a mean-preserving increase in risk *vis-à-vis* F_1. The result in the text follows from the fact that max $(0, y)$ is a convex function and the following standard theorem: a mean-preserving increase in spread from F_1 to F_2 yields $E_1h(y) \leq E_2h(y)$ for any convex function h.

to pay a premium for a private F_π that was riskier than the "public" distribution. This risk-favoring behavior occurs because loss is limited to K and the riskier F_π provides an increased chance of favorable outcomes. The same result is obtained in the theory of option pricing, where a mean-preserving increase in the riskiness of the underlying security leads to an increase in the value of an existing (call) option.

4. A model with fixed capacity

■ The atomistic model is simple because each firm's decision depends only upon the immediate market price. In the general nonatomistic case the optimal entry strategy is impossibly complex, as it depends upon the results achieved by every incumbent. In this section we present a model with major indivisibilities that is still tractable and which presents interesting insights into the selection-based equilibria that uncertain imitability generates. We assume, as before, that there is a sunk cost $K > 0$ associated with each entry attempt. Further, we assume that the firm's marginal cost function is nonincreasing and that each firm faces the same constraint on its maximum output rate. Thus, firms either produce at their full capacity rate or not at all. We model uncertain imitability by simply letting X, the entrant's total cost at full capacity output, be a random variable with distribution F and density f.[15] Each new entrant receives a draw from F that is independent of past draws.

Consider the very simple case in which there can be at most N firms in the industry. Such restrictions might arise, for example, in television broadcasting and taxicab franchises. Shortly, this restriction will be relaxed, and it will be seen that the formal properties of the model remain unchanged. Because these firms always operate at the same full capacity scale, an inverse demand function can be written as a direct relationship between the number n of firms in the industry and the revenues received by each: $R_n = D(n)$. With N firms, each receives a fixed stream of revenues, R_N. If there are fewer than N firms in the industry, the vacant positions are filled on a (arbitrary) first-come first-served basis. When all N positions are filled, entry can still occur if the least efficient firm is displaced. This occurs if a firm attempts entry and displays a level of costs less than C_N, the costs experienced by the *least efficient* of the N firms. The firm that is "bumped" goes out of business, receiving no further revenues, and the position of "least efficient" is filled by another firm. (This mechanistic displacement process is imposed for simplicity in exposition. With a freely varying number of firms, the least efficient firm is displaced only when its cost C_N exceeds R_N, its revenue, for then its profit is negative and it drops out of the industry voluntarily.)

Intuitively, it should be clear that firms will enter, displacing the least efficient, until the least efficient firm has costs low enough to deter further rational entry attempts. The analytic problem is to prove that such a stable equilibrium exists and to derive its properties.

Suppose that in an industry of N firms the least efficient has cost z. To enter this industry successfully a firm would have to achieve a level of cost low enough to displace the incumbent with cost z and to deter any subsequent attempts by others to displace it should it ever become the least efficient firm. To place bounds on the payoff function, define $V(z)$ to be the expected value of attempting entry into such an industry *when any further entry is prohibited.* If the entrant's cost x is less than z, it displaces the least efficient incumbent firm and receives a guaranteed profit stream $R_N - x$; if its cost exceeds z, it gives up its entry attempt. Clearly,

$$V(z) = -K + (1/r) \int_{-\infty}^{z} (R_N - x)f(x)dx, \qquad z \leq R_N . \tag{4}$$

[15] In contrast to the atomistic model of Section 3, larger observed values of X are not desirable.

Now if $V(R_N) < 0$, firms will never enter the industry, because even the guarantee of perpetual residence does not suffice to make entry a breakeven proposition. Consequently, we assume the economic feasibility condition $V(R_N) \geq 0$.

As long as $E|X| < \infty$ and $K > 0$, we can be sure that $V(z) < 0$ for z sufficiently small, insuring that entry-deterring values of z exist. Coupling this fact with $V(R_N) \geq 0$, we can see that V will possess a unique solution Z to $V(z) = 0$ if and only if f is strictly positive in a neighborhood of Z. Without loss of generality we shall simplify the exposition and make this assumption so that Z is indeed unique.[16]

Theorem 1. Firms will continue to enter the industry as long as C_N is at or above Z. Moreover, Z completely characterizes the equilibrium in that entry ceases once C_N falls below Z.

Proof. Because V is an upper bound on the expected value of entry, attempts to enter the industry will never occur if $V(C_N) < 0$, and extant firms in such an industry will never be displaced. Thus, $V(z)$ is precisely equal to the expected value of attempted entry for all $z < Z$, and no entry occurs after C_N falls below Z.

Consider a prospective entrant when $C_N \geq Z$. Since firms with costs less than Z are never displaced, the expected value of entry is at least $V(x)$ for all $x < Z$. Consequently, $\sup\{V(x): x < Z\} = 0$ is a lower bound on the expected value of entry and entry must occur whenever $C_N \geq Z$. *Q.E.D.*

A number of interesting properties of the industry and its evolution follow from our characterization of the free-entry equilibrium. First, the probability that any given entry attempt will result in a permanent position in the industry is simply $F(Z)$. The total number of attempts at entry is a negative binomial random variable with parameters N and $F(Z)$. The expected number of attempted entries is $N/F(Z)$, and $N[1 - F(Z)]/F(Z)$ is the expected number of unsuccessful attempts.

The cost of a firm in the mature industry is a random variable X_Z with density $f(x)/F(Z)$, for $x < Z$ and zero otherwise. The mean and variance of the survivor's costs are given by $\mu_Z = EX_Z$ and $\sigma_Z^2 = E(X_Z - \mu_Z)^2$, respectively. From (4) and the equilibrium condition $V(Z) = 0$ we have

$$rK = \int_{-\infty}^{Z} (R_N - x)f(x)dx = F(Z)[R_N - \mu_Z]. \tag{5}$$

As in the atomistic case, the survivors earn an expected stream of rents. Using (5),

$$R_s = R_N - \mu_Z - rK = rK[1 - F(Z)]/F(Z), \tag{6}$$

so (6) exhibits the same relationship between rents and the probability of successful entry as does (3). The variance of these rents is simply σ_Z^2. Applying the equilibrium condition $V(Z) = 0$ to (4) reveals that the entry blocking cost Z increases with r, K, and $1/R_N$; increases in Z lead in turn to increases in μ_Z and $F(Z)$. Coupling this with (6), we see that increases in r, K, and $1/R_N$ lead to decreases in the survivor's rents, R_s.

Because entry activity depends on the cost of the least efficient firm, *once there are N firms in the industry any additional (successful) entry will increase the average profitability.* Under conditions of uncertain imitability, high levels of profitability can signal difficult-to-replicate levels of efficiency and act to deter rather than induce entry.

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[16] If this assumption does not hold, merely define Z to be the smallest solution to $V(z) = 0$; the existence of a density ensures that V is continuous. Finally, if F does not have a density, define Z by

$$Z = \inf\{x: V(x) \geq 0\}$$

and assume that $\lim_{x \to Z^-} V(x) = 0$. This guarantees that the proof of Theorem 1 goes through.

□ **A freely varying number of firms.** To relax the assumption of a fixed N, we must allow the interaction between the demand function and the entry process to determine the number of firms in equilibrium. Recall that $R_n = D(n)$ is the revenue received by each firm when there are n firms in the industry. By analogy with (4), we define the profit function V_n by

$$V_n(z) = -K + (1/r)\int_{-\infty}^{z} (R_n - x)dF(x) \qquad z \leq R_n, \qquad n = 1, 2, \ldots. \tag{7}$$

Economic feasibility for the entry of n firms requires that $V_n(R_n) \geq 0$. As the number of firms increases, the revenues available to each decrease; accordingly, denote by N the largest n for which $V_n(R_n) \geq 0$ so $V_{N+1}(R_{N+1}) < 0$. This N is appropriately referred to as the natural industry size; at least N firms will enter the industry, and no additional firms will enter if each incumbent firm has cost less than Z_N, the solution to $V_N(z) = 0$. Interestingly, the free-entry equilibrium is not necessarily determined by Z_N.

Theorem 2. In equilibrium there are N firms each with cost no greater than

$$Z \equiv \max \{Z_N, R_{N+1}\}.$$

Moreover, Z completely characterizes the equilibrium in that entry ceases when C_N falls below Z.

Proof. Let N, C_N, and X be the number of firms present, the cost of the least efficient firm, and the cost for the next entrant should it decide to attempt entry, and assume that $C_N > Z_N$. There are two cases to consider: $C_N > R_{N+1}$ and $C_N \leq R_{N+1}$. In the first case suppose that $X \leq Z_N$. Then the firm with cost C_N will be forced out of the industry, for otherwise its profit rate would be $R_{N+1} - C_N < 0$. Consequently, a potential entrant does in fact enter, for it will receive an expected profit of at least $V_N(Z_N) = 0$ upon entry.

In the second case $R_{N+1} - C_N \geq 0$ so that the potential entrant will not dislodge the C_N firm, even if $X \leq Z_N$. Therefore, $V_{N+1}(R_{N+1}) < 0$ is the potential entrant's expected profit and, accordingly, it will not choose to enter, and the C_N firm will never be dislodged. *Q.E.D.*

In this model entrants have an *ex ante* expectation of rents, with early entrants expecting larger amounts of surplus profit. The reason is that the possibility of not being able to dislodge an inefficient firm (one whose cost lies between Z_N and R_{N+1}) drives a wedge between the zero-profits condition $V_N(Z_N) = 0$ and the no-entry condition $V_N(Z) = 0$. Consequently, each firm facing $C_N > Z$ *expects* a discounted surplus profit of $V_N(Z) > 0$ when $Z > Z_N$, and $V_N(Z) = 0$ when $Z = Z_N$.

These additional expected rents attach value to the opportunity to attempt entry, and, given a queue of potential entrants, we can compute the premium P_k associated with the kth position in the queue. Since the probability that an entry attempt will be successful is independent of k, the queue position L of the last firm to attempt entry is a negative binomial random variable with parameters N and $F(Z)$. Clearly P_k decreases in k (strictly for $k > N$), and

$$P_k = V_N(Z)P[L \geq k], \qquad k = 1, 2, \ldots. \tag{8}$$

Summing over all positions in the queue gives P_I, the total discounted value of expected rents for the industry:

$$P_I = V_N(Z) \sum_{k=1}^{\infty} P[L \geq k] = V_N(Z)E(L) = V_N(Z)N/F(Z).$$

It is evident that it is the indivisibility condition which creates the rents and these premiums. With atomistic firms the wedge must disappear.

5. The case of scale-based uncertainty

■ In the case of fixed-size firms, the optimal entry policy depended upon only a single state variable: the cost of the least efficient firm. Are there more general nonatomistic cases in which globally fully informed entry halts if and only if some state descriptor exceeds or falls below a critical value? If such a policy is to be optimal, it must be the case that a single variable is sufficient to describe the industry. One way to accomplish this is to restrict b, the carrier of uncertain imitability, to be a scale parameter.

To begin, let A be any strictly increasing cost function exhibiting a U-shaped average cost curve. We attain the appropriate scaling by assuming that

$$T(q, b) = bA(q/b) \tag{9}$$

so that

$$T'(q, b) = A'(q/b) \tag{10}$$

whenever $b > 0$. Thus both the marginal and average cost functions are scaled by b. (If $b = 0$, let $T(q, 0)$ be infinite or zero as $q > 0$ or $q = 0$.) We assume that A' is strictly increasing and continuous so that it has a continuous strictly increasing inverse labeled h. Because each firm acts as a price-taker, the firm with scale parameter b confronting price p maximizes its profits by (setting $A'(q/b) = p$) producing

$$q = bh(p). \tag{11}$$

In view of (11), and because the rate of output which minimizes average cost is also linear in b, it is clearly appropriate to refer to b as the *size* of the firm. Moreover, if b_i is the size of the ith firm, then Q, the total industry output at price p, is simply

$$Q = h(p) \sum b_i \equiv h(p)B, \tag{12}$$

and $B \equiv \Sigma\, b_i$ must be interpreted as the industry size.

We assume that the industry demand function is continuous and downward sloping. Coupling this assumption with (12) and the fact that h is continuous and strictly increasing yields the existence of p_B, the unique market-clearing price associated with any industry of size B. Furthermore, p_B is continuous and strictly decreasing.

The stream of profits accruing to a firm of size b when confronting the price p is given by

$$\pi = b[ph(p) - A(h(p))]. \tag{13}$$

Consequently, defining H by

$$H(B) = p_B h(p_B) - A(h(p_B)), \tag{14}$$

π admits the representation

$$\pi = bH(B). \tag{15}$$

Of course, H is strictly decreasing[17] and continuous. It is particularly helpful to interpret $H(B)$ as the "operating profits" of a firm of unit size in an industry of size B.

Because (9) permits the inclusion of scaled fixed costs, π may become negative for sufficiently large B. From (15), negative profits for one firm imply negative profits for the entire industry. While our problem formulation allows $\pi < 0$, it cannot permit firms to withdraw from the industry once they have entered. Were withdrawals permitted, the possibility of each extant firm's withdrawing would have to be considered by a prospective entrant, thereby invalidating the concept of a single summary state variable. Consequently, we assume either that negative profits are borne indefinitely (e.g., an unavoidable lease commitment or a noncash amortization) or that the cost and demand conditions do not

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[17] Noticing that $A'(h(p)) = p$ yields $dH/dp = h(p) > 0$.

admit a solution for which $H < 0$.[18] With this condition, the state of the industry is summarized by the value of B.

Now consider a potential entrant's decision and, as per (1) and (4), define $V(B)$ to be the expected discounted value of entry when the current industry size is B and no additional entrants will follow. Clearly, V satisfies

$$V(B) = -K + (1/r)\int_0^\infty bH(B+b)dF(b). \tag{16}$$

Assume that $EX < \infty$, then $V(B) < \infty$, and V is clearly a continuous decreasing function as H is continuous decreasing. Assuming the economic feasibility condition $V(0) > 0$, there will be, therefore, a unique solution B^* to

$$V(B) = 0 \tag{17}$$

provided $V(B) < 0$ for B sufficiently large. This last condition is met when

$$\lim_{B\to\infty} H(B) \le 0,$$

which is equivalent to requiring that profits vanish as arbitrarily many firms enter; we shall make this reasonable assumption.

□ **The existence of a unique equilibrium B^*.** If the industry's size B is greater than B^*, $V(B) < 0$ and no firm would enter even if further entry were prohibited. Thus we are guaranteed that no entry will take place if the industry's size exceeds B^*. But is the converse true? That is, will entry always occur when the industry size is less than B^*? The answer is no. To see this define $\Pi(B)$ to be the expected value of a firm that enters when the industry size is B and consider the following example.

Example 1. Let X take the values 1 and 7 with probability ⅔ and ⅓, respectively, let the demand function be $p = 20r/q$, and set $K = 3$, and let $T(q, b) = r(1 + q^2/2b)$ so that $\pi = rbp^2/2 - r(K+1)$ and $H(B) = rp_B^2/2 - r = 10r/B - r$. Then

$$V(B) = -(K+1) + \frac{2}{3}\frac{10}{B+1} + \frac{7}{3}\frac{10}{B+7}.$$

Consequently, $B^* = 3$, and entry ceases for $B > 3$. Moreover, the fact that X is at least one implies that $\Pi(B) = V(B) > 0$ whenever $2 < B < 3$. However, when $B = 2$, two firms might enter, and the expected value is negative. That is,

$$\Pi(2) = -4 + \frac{7}{3}\frac{10}{9} + \frac{2}{3}\left[\frac{2}{3}\frac{10}{4} + \frac{1}{3}\frac{10}{10}\right] = -4 + \frac{106}{27} = -\frac{2}{27} < 0.$$

Hence, entry does not occur at $B = 2 < B^*$. *Q.E.D.*

The example illustrates what might be called a "first mover" disadvantage. The potential entrant confronting $B = 2$ faces double jeopardy—a low b yields not only a small firm with low profits but also an industry size B just short of the level required to halt further entry, thereby inducing an equilibrium price with small expected value. Of particular interest is the fact that entry will never occur, even though there are profits available to a cartel. The "second mover," confronting $B = 3$, is guaranteed that there will be no further entry and therefore faces a more favorable situation. Of course, if the first mover is rational and does not enter, the second mover will never get a chance.[19]

[18] The possibility of exit from the industry because of negative profits stems from the existence of a fixed cost F as exhibited, for example, in $T(q, b) = bF + q^2/2b$. In the short run the fixed cost F is unavoidable, and there will be no exit. In the longer run, however, all costs are variable, and exit could occur.

[19] The first-mover disadvantage might evaporate if an entrant were awarded the right-of-first-refusal with respect to the immediately ensuing entry opportunity. In Example 1 the expected profit accruing to the entrant at $B = 2$ remains negative. However, examples can be constructed in which this right induces entry.

It is clear upon reflection that the decreasing function V is such that $V(B) = \Pi(B)$ whenever $B > B^*$. However, Example 1 reveals not only that Π need not be monotonic as is V, but more importantly that Π need not be positive on $[0, B^*)$, so entry need not occur for all industry sizes less than B^*.

Fortunately, we can circumvent this problem and guarantee that entry ceases if and only if the industry size exceeds B^*; this guarantee is effected by placing a modest restriction on the distribution function F. The actual industry size will, of course, depend upon the size of the final entrant. Thus, the equilibrium industry size is a random variable (described below).

In establishing that B^* is the unique stopping point, we begin by introducing some notation and concepts from renewal theory.

Let $\langle X_i \rangle$ be a sequence of independent random variables, each with distribution F, $S_k = \sum_{i=1}^{k} X_i$, and $\{N(t): t \geq 0\}$ the associated renewal process so that

$$N(t) = \max \{n: S_n \leq t\}.$$

Making the identification between S_k and B, it may be seen that $N(t)$ is the number of firms in the industry when $B = t$. We also introduce the functions $m(t) = E\{N(t)\}$ and $Y(t)$, the excess life at t, where

$$Y(t) = S_{N(t)+1} - t. \tag{18}$$

To understand the excess life in terms of our model of entry, note that the actual industry sizes observed during entry are S_0, S_1, etc. Suppose the current industry size is $B = S_j$ and we name a larger industry size $B + t$. Entry now continues and the first $S_{n+j} \geq B + t$ occurs at $n = N(t) + 1$; $Y(t)$ is the amount by which the new industry size exceeds $B + t$.

If it were true (as we shall prove in Theorem 3) that firms continue to enter the industry until its size exceeds B^*, then the random variable M_B, the equilibrium industry size when the current industry size is B, would be given by

$$M_B = B^* + Y(B^* - B), \qquad B \leq B^*. \tag{19}$$

Letting G_B denote the distribution of M_B and adopting the convention that G_B is a unit mass at B for $B > B^*$, the expected profit of a firm that enters when the industry size is B would satisfy

$$\Pi(B) = -K + \frac{1}{r} \int_0^\infty \int_{B^*}^\infty bH(z)dG_{B+b}(z)dF(b), \qquad B \leq B^*. \tag{20}$$

To ensure that M_B is indeed the equilibrium industry size, the unique solution B^* to $V(B) = 0$ should represent the worst possible case for the profits of a potential entrant. Suppose $B^* = 1000$ and $P(X = 1) = .999 = 1 - P(X = 10^6)$, then $P(M_{B^*} = 1001) = .999$, yet $P(M_0 = 1001) \approx e^{-1}$ so B^* is not the worst possible case. Thus, some restriction on the distribution of X must be imposed.[20] One obvious way to obtain the desired result is to require the final industry size M_B be stochastically smaller than M_{B^*}.

From (19) we see that M_B will be stochastically smaller than M_{B^*} if and only if $Y(B^* - B)$ is stochastically smaller than X. This leads us to consider the class of distributions known in reliability theory as "new better than used" (NBU). Specifically, the random variable X is said to be NBU if

$$P[X > s] \geq P[X > t + s | X > t], \qquad \text{for all} \qquad s > 0, t > 0, \tag{21a}$$

[20] In Example 1 with $B^* = 3$ we did not have M_2 stochastically smaller than M_3 as $P[M_3 \leq 4] = 6/9$ while $P[M_2 \leq 4] = 4/9$. The reason is that $P[X \leq t + s | X > s]$ was not increasing in s.

or, equivalently,

$$F(t+s) - F(t) \geq F(s)[1 - F(t)], \qquad \text{all} \qquad s > 0, t > 0. \tag{21b}$$

If F is NBU, (21) implies that the probability that X lies in the region $[0, s]$ is never larger than the probability that X lies in the region $[t, t+s]$, given that $X > t$. Using the notation $\bar{F}(t) = 1 - F(t)$, (21) is also equivalent to

$$\bar{F}(s)\bar{F}(t) \geq \bar{F}(t+s). \tag{22}$$

The class NBU is quite large and includes, for example, the exponential, gamma, Weibull, truncated normal, displaced forms of these distributions, and contains all distributions with increasing failure rates.

The following Lemma is needed to establish Theorem 3 in which we prove that when F is NBU entry does indeed continue until the industry size exceeds B^*.

Lemma. If F is NBU, then for each $t > 0$

$$X \text{ is stochastically larger than } Y(t). \tag{23}$$

Proof. Let $Z(t) = t - S_{N(t)}$ have distribution G, and note that

$$P(Y(t) \leq x | Z(t) = s) = [F(s+x) - F(s)]/[1 - F(s)] \geq F(x)$$

so that

$$P(Y(t) \leq x) = \int_0^t P(Y(t) \leq x | Z(t) = s) dG(s) \geq \int_0^t F(x) dG(s) = F(x). \text{ Q.E.D.}$$

Theorem 3. Entry ceases when the industry size exceeds B^*. If X is NBU, then firms continue to enter until the industry size exceeds B^* so B^* fully characterizes the free-entry equilibrium.

Proof. If $\Pi(B) \geq 0$ for each $B < B^*$, then firms will continue to enter the industry until its size exceeds B^* in which case M_B is the free-entry equilibrium industry size and $\Pi(B)$ does indeed satisfy (20). Consequently, we must demonstrate that

$$\Pi(B) \geq V(B^*) = 0$$

for each $B < B^*$.

Fix $B < B^*$, $T > B^*$, $u = B^* - B$, $v = T - B^*$, and consider the indicator function $1_{[0,T]}$. Because the decreasing function $H(B)$ is the limit of an increasing sequence of step functions, the Monotone Convergence theorem can be utilized to reduce our problem to merely verifying (see Appendix A)

$$\int_0^\infty b \int_{B^*}^\infty 1_{[0,T]}(z) dG_{B+b}(z) dF(b) \geq \int_0^\infty 1_{[0,T]}(B^* + b) b dF(b). \text{ Q.E.D.} \tag{24}$$

At first blush it seems reasonable to believe that Theorem 3 is a simple and direct consequence of the Lemma. The complication arises not so much from the fact that M_B—and hence the market-clearing price—depends upon the new entrant's size, but rather because the new entrant's profit is the product of his size and the market-clearing price. This problem persists even when F is exponential so that M_B is independent of B (i.e., M_B has the same distribution as $B^* + X_1$).

It should be evident that $K > 0$ is required in this model when $H \geq 0$ or there would be no impediment to entry whatsoever and entry would never cease; also $V(0) < 0$ for K sufficiently large, and the industry never exists. Unless B^* characterizes the free-entry equilibrium, it is difficult to say anything in general about the impact of changes in K on the equilibrium. However, if X is NBU, we can show that a decrease in K will lead to an increase in B^* and, concomitantly, a decrease in the free-entry equilibrium price. In particular, let B_K denote the solution to (17) when $K > 0$ is the entry cost and let the random variable P_K denote the free-entry equilibrium price.

Theorem 4. If F is NBU, then the minimal industry size B_K is strictly decreasing in K and the free-entry equilibrium price P_K is nondecreasing in K.

Proof. It is evident from (16) and (17) that B_K satisfies

$$rK = \int_0^\infty bH(B_K + b)dF(b).$$

Since H is a strictly decreasing function, B_K is strictly decreasing in K. Next, let ξ_K denote the free-entry equilibrium industry size when the entry cost is $K > 0$. It follows from (19) that

$$\xi_K = S_{N(B_K)+1} .$$

Since B_K is strictly decreasing in K, $\xi_K(\omega)$ is a nonincreasing function of K for *each* point ω in the sample space, and the nondecreasing nature of P_K now follows from the fact that the demand function is downward sloping and h increasing. *Q.E.D.*

The impact on the equilibrium of changes in the distribution of X, particularly mean-preserving increases in risk, is not predictable without further specification of the cost and/or demand functions. Still, it may be noted that since B^* solves

$$rK = \int_0^\infty bH(B + b)dF(b),$$

B^* will decrease with mean-preserving increases in the risk of F provided $bH(B + b)$ is concave in b for each fixed $B \geq 0$. Since H is decreasing, it suffices that $H(B + b)$ be a concave function of b.

Example 2. Let the demand function have constant elasticity ϵ so that $Q = Dp^{-\epsilon}$, and let $T(q, b) = q^2/2b$. Then $bH(B + b) = b(B + b)^{-2/(1+\epsilon)}$ is concave if and only if $2 - [b/(b + B)][(3 + \epsilon)/(1 + \epsilon)] \geq 0$. Consequently, we can conclude that $\epsilon \geq 1$ suffices to ensure that B^* decreases with mean-preserving increases in the riskiness of X. *Q.E.D.*

□ **Economic rents.** Despite price-taking behavior and a perfectly scaled family of firms, there are positive expected rents in equilibrium. These rents are due to a special type of indivisibility—the inability of the final industry entrant to control its size.

Assuming that B^* characterizes the free-entry equilibrium, the expected rent stream $\bar{R}$ accruing to the industry satisfies

$$\bar{R} = -rK[m(B^*) + 1] + \int_0^\infty xH(x)dG_0(x), \tag{25}$$

where m is the renewal function for $\{N(t): t \geq 0\}$. This expression does not yield any immediate insight into the sign of $\bar{R}$. If F is NBU, however, the proof of Theorem 3 can be employed (Lippman and Rumelt, 1980b) to show that $\Pi(B) > \Pi(B^*) = 0$ for $B < B^*$, thereby establishing that each firm in the industry receives a positive expected surplus profit.

Theorem 5. If F is NBU, then $\bar{R} > 0$.

If, as asserted, it is the indivisibility of firms that enables rents to exist, rents will disappear with atomistic firms. We demonstrate that appropriately scaling down firm size causes the minimal industry size B^* to increase until the rents disappear.

To begin, let the t-scale problem be defined by the random firm size X_t, entry cost K_t, and total cost function T_t, where

$$X_t = X/t, \qquad K_t = K/t, \qquad \text{and} \qquad T_t(q, b) = T(q, b). \tag{26}$$

It is important to observe that replacing one firm of size b whose production is q in the original problem by t firms, each of size b/t, in the t-scale problem yields a total output of

$$tq_t = t\left\{\frac{b}{t}\,h(p)\right\} = bh(p) = q, \tag{27}$$

total entry costs of

$$tK_t = K, \tag{28}$$

and by (9) and (27) total costs of

$$tT_t(q_t, b/t) = tT(q/t, b/t) = t\,\frac{b}{t}\,A(q/b) = T(q, b). \tag{29}$$

Thus, the problem has been appropriately scaled.

Next, define the random variable R_t to be the total industry rent stream in the t-scale problem and denote its expected value by $\bar{R}_t$. In addition, denote the solution of (17) for the t-scale problem by B_t so that B_t is the minimal industry size. As t grows larger, it is obvious that individual firms and their rents become smaller. The following theorem assures us that as t grows without bound, B_t increases to a finite limit, R_t approaches zero with probability one, and $\bar{R}_t$ also vanishes in the limit. (See Lippman and Rumelt (1980b) for the proof.)

Theorem 6. If F is NBU, then the minimal industry size B_t for the t-scale problem increases in t, $\lim_{t\to\infty} B_t = B_\infty < \infty$, and B_∞ is the unique solution to

$$rK = E(X)H(B_\infty). \tag{30}$$

Furthermore,

$$R_t \to 0 \quad \text{as} \quad t \to \infty \quad \text{with probability 1} \tag{31}$$

and

$$\bar{R}_t \to 0 \quad \text{as} \quad t \to \infty. \tag{32}$$

Rents disappear under true atomism, but what happens if our nonatomistic firms behave as if they were atomistic? Suppose, as before, that each firm is a price-taker in that it takes price as marginal revenue. However, now the firm presumes (incorrectly) that its production has no impact on the market-clearing price or on the decisions of other potential entrants. With this pure price-taking behavior, entry will occur until $H(B)E(X) = rK$, which by (30) is the definition of B_∞. Because $B_\infty > B^*$, the industry will be overcrowded, and each entrant's actual expected profit will be negative (because the firm's expected profit is bounded above by $V(B_\infty) < V(B^*) = 0$). The resulting inefficiency emanates from the firm's failure to account for the negative externality associated with entry.

6. Concentration, profitability, and risk

■ The profitability of firms and industries is a major concern in industrial organization. In the "structure-conduct-performance" paradigm, industry structure (concentration) has a causal effect (mediated by firm behavior) on industry profitability. By contrast, in the model using scale-based uncertainty, structure and profitability are the joint outcomes of an underlying stochastic process. This section examines some of the connections between firm profit rates, industry profitability, and concentration implicit in the model defined and analyzed in Section 5. We begin by discussing the general character of the equilibrium and then investigate Monte Carlo solutions to particular specifications.

Defining a firm's *rent margin* w as the ratio of surplus profits, or rents, to revenues, it is clear from (11) and (15) that the rent margin for a firm of size b is given by

$$w = [H(B) - rK/b]/[ph(p)], \tag{33}$$

and that the *industry rent margin* W is

$$W = [H(B) - rK/(B/N)]/[ph(p)], \tag{34}$$

where N is the number of firms in the industry. The industry rent margin depends upon both the final industry size B and the number N of firms. Other things being equal (i.e., given the industry size B), industries with fewer firms will appear more profitable. In interpreting the size of W in what follows it is important to recall that there are no failed entry attempts in the scale-based uncertainty model. Consequently, W is purely a function of the indivisibilities and uncertainty in this model and does not include "survivor's rent" components as in (3) and (6).

Comparing (33) and (34), it can be seen that $w = W$ when $b = B/N$. Thus, firm rent margin is increasing in b, displaying a deterministic "market share" effect. Obviously, it is incorrect to interpret this association between relative size and profitability as evidence of market power; a pure scale economies explanation would also be incorrect. The association arises because the measure of profitability includes the ratio of a fixed input to an uncertain outcome. The possible existence of this mechanism calls into question the practice of taking cross sectional associations between profitability and market share as demonstrating market power or marginal bargaining power within collusive oligopolies (Kwoka, 1979). Such attributions do not appear appropriate unless the effects of phenomena like uncertain imitability have been controlled or ruled out.[21]

Does the model predict a positive association between concentration and profitability? Not unambiguously. Concentration is usually measured by either the proportion of output due to the largest n firms or the Herfindahl index, which is the sum of the squared market shares of all firms in the industry. Both measures mix two elements of concentration: the number of firms and the variance in firm size. We expect concentration to increase as the number of firms decreases and as the variance in firm size increases. Looking only at the number of firms, it is evident from (34) that W increases with falling N. However, in equilibrium both the industry size M_0 and the number N of firms are random variables; moreover, they are not independent. An especially large final entrant, for example, will tend to both reduce N and increase the final industry size, so that the net impact on W and on concentration is ambiguous. Consequently, we cannot predict in advance the sign of the association across sample paths between concentration and profitability.

Turning to the influence of mean-preserving increases in the spread of F, it should be clear the variance of market shares will increase. Nevertheless, concentration may still fall as increases in the riskiness of F can lead to increases in the expected number of firms. (To see this, consider the case in which F is a point mass just above B^*/j so that in equilibrium there are always j firms in the industry.)

Because the model's structure-performance implications are ambiguous short of a complete specification, we turn to the study of a particular case. The model specification we investigate is quite simple and was chosen for clarity rather than realism.

The industry demand function has unit-elasticity: $Q = J/p$; the firm's cost function is quadratic: $T(q, b) = q^2/2b$; the distribution of X is gamma with parameters s and λ so that $EX = s/\lambda$ and $\text{Var } X = s/\lambda^2$. In what follows we keep $s = \lambda$ so that increases in s correspond to mean-preserving decreases in risk, and, concomitantly, a decrease in $\text{Var } X$. Defining the dimensionless constant α as

$$\alpha = 2rK/J, \tag{35}$$

[21] In a study addressed at this question, Rumelt and Wensley (1981) analyzed the associations between *changes* in profitability and *changes* in market share and found that the strong initial association disappeared when controls for the stochastic, or unanticipated, components of output growth were included.

it is a simple matter to apply the model specification to (16) and (17) to show that B^* solves

$$\alpha = E\{X/(X + B)\} \tag{36}$$

and that

$$H(B) = J/2B, \tag{37}$$

$$W = [1 - N\alpha]/2 \tag{38}$$

$$w = [1 - \alpha B/b]/2. \tag{39}$$

To measure concentration we simply chose S_{max}, the market share of the largest firm (the one-firm concentration ratio). Given a model specification, the equilibrium values of S_{max} and W are random variables; we took their correlation (hereafter *COR*) as one indicator of a relationship between concentration and profitability. Additionally, connection between concentration and profitability can arise *across* model specifications, so we also investigate parallel movements in ES_{max} and EW with changes in α and Var X.

Calculations were performed to determine the properties of the equilibria for three different values of α; in each case we let Var X range from zero (certainty) to its maximum value of 1 ($s = 1$ yields the exponential distribution). The value of B^* was calculated by solving (36) and values for EN, EW, ES_{max}, and *COR* were obtained from a mixture of direct calculations and Monte Carlo methods (see Appendix B for details).

The results are shown in Table 1 and the relationships between ES_1 and EW are graphed in Figure 1. *In each case COR was large and positive.* Furthermore, *COR was essentially independent of Var X*, indicating that it arose solely from the effect of changes in N on both concentration and the rent margin.

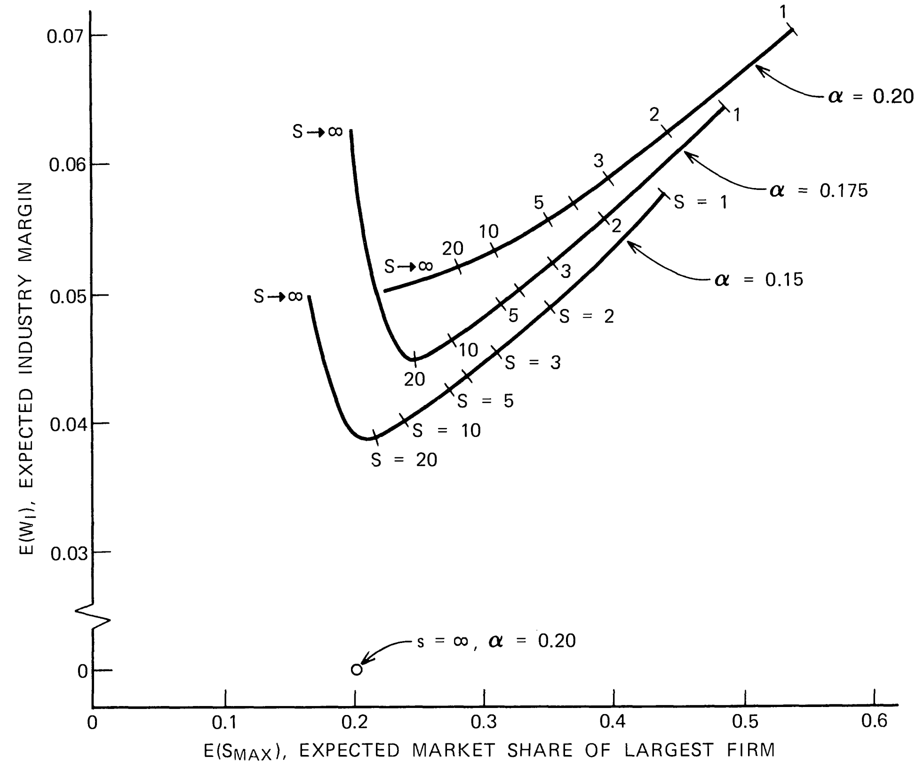
Turning to the impact of changes in the variance of X, Figure 1 shows a clear *monotonic increase in concentration with rises in Var X* (decreasing s). For $s < 20$ there

TABLE 1 **Numerical Results for Example Model Specification**

			$\alpha = .15$			
s	1	2	5	10	20	$s \to \infty$
Var X	1	.5	.2	.1	.05	0
B^*	4.900	5.267	5.395	5.583	5.625	5.667
EN	5.900	6.017	6.062	6.133	6.149	6.000
EW	.0575	.0488	.0454	.0400	.0388	.0500
ES_{max}	.438	.349	.310	.238	.213	.167
COR	.61	.63	.63	.63	.62	—
			$\alpha = .175$			
s	1	2	5	10	20	$s \to \infty$
Var X	1	.5	.2	.1	.05	0
B^*	3.982	4.329	4.554	4.633	4.674	4.714
EN	4.982	5.079	5.154	5.183	5.196	5.000
EW	.0641	.0556	.0490	.0465	.0453	.0625
ES_{max}	.487	.394	.315	.276	.247	.200
COR	.66	.68	.64	.68	.69	—
			$\alpha = .2$			
s	1	2	5	10	20	$s \to \infty$
Var X	1	.5	.2	.1	.05	0
B^*	3.299	3.623	3.845	3.922	3.961	4.000
EN	4.299	4.379	4.445	4.471	4.482	4.500
EW	.0701	.0621	.0555	.0529	.0518	.0500
ES_{max}	.537	.441	.353	.310	.281	.225
COR	.66	.68	.67	.69	.74	—

FIGURE 1

E(W_I) VS. E(S_{MAX}) FOR THREE CASES



is additionally a clear increase in the expected industry rent margin as risk increases; the model shows concentration and profitability both moving positively with risk for high levels of Var X. However, neither EN nor EW is necessarily monotonic with respect to increases in Var X. In the case $\alpha = .15$ the value of EW is minimum at $s = 26$, with either higher or lower levels of variance in X leading to a larger expected rent margin. At $s = 26$ we expect 6.15 firms in equilibrium; movements away from this point in either direction *decrease* the expected number of entrants and thereby raise the expected rent margin. As we move to greater certainty, the problem tends towards an integer solution ($N = 6$); more uncertainty raises the chance of a large last entrant and thereby reduces B^* as well as the expected number of entrants.

The parameter α increases with the significance of the fixed-entry fee relative to the size of the industry and measures the importance of indivisibilities. For high levels of uncertainty, EN decreases smoothly as α increases, and EW is a steadily increasing function of α. However, as Var X becomes small, the effects of the indivisibilities are magnified and EW is no longer monotonic in α. In the case of certainty, rents jump upward discontinuously where $1/\alpha$ is an integer and decrease smoothly in α everywhere else. This "sawtooth" relationship between α and EW under certainty explains the behavior of the curves in Figure 1, where EW is larger for $\alpha = .175$ than for $\alpha = .15$ or $\alpha = .2$.

An interesting detail arises with regard to the case $\alpha = .2$. Under certainty, $B^* = 4$, and straightforward application of (38) indicates a zero-rent solution involving five firms. Nevertheless, the limiting case of our model as Var X approaches zero is not zero-rent. Instead, we find $EN = 4.5$ and $EW = .05$, so that the solution in the limit as Var X approaches zero is not equivalent to the solution obtained by setting Var X equal to zero. To see why this happens, consider the case in which $EX = 1$ and the variance of X is very small but not zero. About one-half of the time the first four entrants will produce a B

that falls just short of $B^* = 4$ and a fifth firm will enter. The other half of the time the first four firms will just overshoot B^*, and there is no further entry. On average, 4.5 firms will enter; this continues to occur as we allow Var X to approach zero. Whereas under certainty there is an exact zero-rent solution whenever $1/\alpha$ is an integer, in all such cases the limiting solution produced by our model is $W = \alpha/4$, not zero-rent.

The numerical results demonstrate that even an artificially simple specification of this model can exhibit a surprisingly complex solution structure. We found concentration and industry rent margin to be correlated across sample paths in all cases. Expected concentration rose with increases in uncertain imitability. The expected industry rent margin, however, showed an interior minimum at a mid-level of Var X, rising from the minimum with both increases and decreases in uncertain imitability.

7. Concluding remarks

■ Uncertain imitability is a theory explaining the origin and persistence of interfirm differences in efficiency. In this article we have shown how it may be used to build models of free-entry industry equilibria for cases in which causal ambiguity and factor immobility figure importantly.

Some of our results are intriguingly counterintuitive when viewed from the perspective of classical theory: atomistic price-takers may display significant rents in a free-entry equilibrium; social welfare can increase with mean-preserving increases in risk; in the absence of entry barriers, the propensity to enter an industry may decline with increases in the industry's profitability. These conclusions are the straightforward consequence of uncertainty in the creation of production functions and an equilibrium process that emphasizes selection rather than individual adaptation.

We have presented numerical results which suggest that industries characterized by uncertain imitability may exhibit associations between market share and profitability, between concentration and profitability, and between average profitability and its variance among firms. Inasmuch as these results do not flow from the exercise of market power, they suggest an internally consistent alternative interpretation of associations of this type. In principle, the case of uncertain imitability is distinguishable from monopoly or collusive oligopoly by the observed persistent *dispersion* in profit rates among extant firms.

Extension of the solution concepts presented here to other methods of parameterizing cost functions is very difficult in all but the atomistic case. The optimal policy depends in all too vivid detail upon the characteristics of each firm in the market. One obvious approach is to abandon the assumption of globally optimal policies and to assume that entry occurs according to some reasonable heuristic. Another fruitful line of inquiry involves the application of uncertain imitability to monopolistic competition.

Appendix A

Proof of theorem 3

■ Working on the left-hand side of (24), we have

$$\int_0^u b \int_{B^*}^\infty 1_{[0,T]}(z)dG_{B+b}(z)dF(b) + \int_u^\infty b \int_{B^*}^\infty 1_{[0,T]}(z)dG_{B+b}(z)dF(b) =$$

$$\int_0^u bP(Y(u-b) \leq T - B^*)dF(b) + \int_u^{u+v} bdF(b) \geq P(X \leq v) \int_0^u bdF(b) + \int_u^{u+v} bdF(b),$$

where the inequality follows from the Lemma. Similarly, the right-hand side of (24) equals $\int_0^v bdF(b)$. Consequently, it suffices to verify

$$P(X \leq v) \int_0^u bdF(b) + \int_u^{u+v} bdF(b) - \int_0^v bdF(b) \geq 0. \quad \text{(A1)}$$

Using integration by parts, which gives, for example,

$$\int_0^v t dF(t) = \int_0^v \bar{F}(t)dt - v\bar{F}(v),$$

with $\bar{F}(t) = 1 - F(t)$, (A1) reduces to

$$v\bar{F}(v) - \bar{F}(v) \int_0^u \bar{F}(t)dt - (u + v)\bar{F}(u + v) + u\bar{F}(u)\bar{F}(v) + \int_v^{u+v} \bar{F}(t)dt \geq 0. \quad \text{(A2)}$$

As X is NBU, $\bar{F}(u + v) \leq \bar{F}(u)\bar{F}(v)$, whence it suffices to show $\xi \geq 0$, where

$$\xi \equiv v\bar{F}(v) - \bar{F}(v) \int_0^u \bar{F}(t)dt - v\bar{F}(u + v) + \int_v^{u+v} \bar{F}(t)dt. \quad \text{(A3)}$$

To verify $\xi \geq 0$ we consider two cases. If $v \leq u$, then write

$$\int_0^u \bar{F}(t)dt = \int_0^v \bar{F}(t)dt + \int_v^u \bar{F}(t)dt \leq v + \int_v^u \bar{F}(t)dt \quad \text{(A4a)}$$

and

$$\int_v^{u+v} \bar{F}(t)dt = \int_v^u \bar{F}(t)dt + \int_u^{u+v} \bar{F}(t)dt \geq \int_v^u \bar{F}(t)dt + v\bar{F}(u + v) \quad \text{(A4b)}$$

so that

$$\xi \geq [1 - \bar{F}(v)] \int_v^u \bar{F}(t) \geq 0.$$

If $v > u$, then write

$$\int_0^u \bar{F}(t)dt \leq u \quad \text{(A4c)}$$

and

$$\int_v^{u+v} \bar{F}(t)dt \geq u\bar{F}(u + v) \quad \text{(A4d)}$$

so that

$$\xi \geq (v - u)[\bar{F}(v) - \bar{F}(u + v)] \geq 0. \text{ Q.E.D.}$$

Appendix B

Calculation methodology

■ For each specification of α and s the corresponding value of B^* was obtained by solving

$$(s - 1)!\, \alpha = \int_0^\infty \frac{(st)^s e^{-st}}{t + b} dt$$

using a Newton-Raphson method applied to values obtained from numerical integration. Given B^*, exact values of EN and EW could be calculated. Because $EX = 1$ in this problem, $EN = m(B^*) + 1$ and, consequently,

$$EM_0 = ES_{N(B^*)+1} = EX[m(B^*) + 1] = m(B^*) + 1,$$

so that EN is also the final industry size and may be calculated as 1 plus the renewal function at B^*. For the exponential case ($s = 1$), $m(t) = st$. For $s = 2$ we have F gamma and $m(t) = t - [1 - \exp(-4t)]/4$. For larger values of s the formula for $m(t)$ has been given by Barlow and Proschan (1965, p. 57) as

$$m(t) = \frac{\lambda t}{s} + \frac{1}{s} \sum_{j=1}^{s-1} \frac{\theta^j}{1 - \theta^j} [1 - e^{-\lambda t(1-\theta^j)}],$$

where

$$\theta = e^{2\pi i/s}.$$

Values for ES_{max} and COR were obtained through Monte Carlo simulation. Given values of α, s, and B^*, several thousand simulated industries were created, with the number of trials adjusted to make the standard errors of the displayed estimates less than one percent of the estimates.

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